**ELEMENTARY LINEAR ALGEBRA – SET 3**

*Analytic geometry on the plane*

1. The triangle is spanned by vectors $\vec{v}, \vec{u}, \vec{w}$. Using vector calculus, express the medians of that triangle in terms of these vectors.

2. The parallelogram is spanned by vectors $\vec{v} = (1,2), \vec{u} = (-3,4)$. Compute the acute angle between the diagonals.

3. The lenghts of vectors $\vec{v}$ and $\vec{u}$ are equal to 3 and 5, respectively. Knowing that $\vec{v} \circ \vec{u} = -2$, compute $(\vec{v} - \vec{u}) \circ (2\vec{v} + 3\vec{b})$.

4. Write an equation of the line through points $P_1 = (2,3)$ and $P_2 = (-3,7)$ in the three forms: directional, slope-intercept and general.

5. If a line $\ell$ through $P = (x_0, y_0)$ has a direction vector $\vec{v} = (a,b)$, then the parametric form of an equation of $\ell$ is given by the system of equations

$$\begin{cases} x &= x_0 + at \\ y &= y_0 + bt \end{cases}$$

   where $t \in \mathbf{R}$ is a parameter. Find the parametric form of an equation of the line through $P = (3,4)$ with a direction vector $\vec{v} = (1,2)$. Then find the parametric form of an equation of the line given in Problem 4.

6. Find the intersection point of the lines with equations given in the parametric form

$$k : \begin{cases} x &= 1 - t \\ y &= 3 + t \end{cases} \quad \text{and} \quad l : \begin{cases} x &= 2s \\ y &= 3 - s \end{cases}$$

7. Find an equation in the general form of the line through $P = (1,2)$ which is parallel to the line with equation $2x + 3y - 1 = 0$.

8. Find an equation in the general form of a line through $P = (1,2)$ which is perpendicular to the line with equation $2x + 3y - 1 = 0$.

9. Find $m$ such that the distance between points $P_1 = (1,0)$ and $P_2 = (m+3,-2)$ is equal to 4.

10. Compute the altitude of the triangle with vertices $A = (0,0), B = (-1,3), C = (2,5)$ through the vertex $C$.

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(most problems are taken from the lists of M. Gewert and Z. Skoczylas)