ELEMENTARY LINEAR ALGEBRA – SET 7

Determinants, systems of linear equations

1. Write the Laplace expansions of the given determinants along indicated rows or columns (do not perform calculations of the determinants)

$$\begin{vmatrix} -1 & 4 & \mathbf{3} \\ -3 & 1 & \mathbf{0} \\ 2 & 5 & \mathbf{-2} \end{vmatrix}, \quad \begin{vmatrix} 1 & 2 & 3 & 0 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{2} \\ 2 & 3 & 3 & 0 \\ 1 & 2 & 3 & 1 \end{vmatrix}, \quad \begin{vmatrix} 1 & 3 & 2 & \mathbf{1} \\ 2 & 4 & -1 & \mathbf{0} \\ -1 & 0 & 2 & \mathbf{0} \\ 3 & 2 & 5 & \mathbf{-1} \end{vmatrix}$$

2. Calculate the determinants

$$\begin{vmatrix} -2 & 4 \\ -3 & 1 \end{vmatrix}, \quad \begin{vmatrix} 1 & 2 & 3 \\ -1 & 1 & -1 \\ 2 & 1 & 3 \end{vmatrix}, \quad \begin{vmatrix} 1 & 0 & 2 & 1 \\ 2 & 1 & -1 & 3 \\ -1 & 0 & 2 & 0 \\ 3 & 2 & 1 & 1 \end{vmatrix}$$

3. Using the properties of the determinants, justify that the following matrices are singular

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ -2 & -4 & -6 \end{vmatrix}, \quad \begin{vmatrix} 1 & 3 & 2 & 1 \\ 4 & 2 & 1 & 3 \\ 3 & 3 & 1 & 2 \\ 0 & 4 & 2 & 0 \end{vmatrix}$$

4. Compute the determinants in Problem 2, using the Gauss algorithm.

5. Using the cofactor formula, compute the inverses of the following matrices:

$$\begin{pmatrix} -2 & 4 \\ -3 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 2 & -1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 4 & 0 \end{pmatrix}$$

6. Using inverse matrices, solve the following matrix equations:

$$(a) \quad \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} \cdot X = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 3 & -1 \end{pmatrix}, \quad (b) \quad \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 2 & -1 \end{pmatrix} \cdot X = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

7. Applying Cramer's Rule to the following systems of equations, compute the indicated unknown:

$$(a) \quad \begin{cases} 2x - y = 0 \\ 3x + 2y = 5 \end{cases}, \text{ unknown } y \quad (b) \quad \begin{cases} x + y + 2z = -1 \\ 2x - y + 2z = -4 \\ 4x + y + 4z = -2 \end{cases}, \text{ unknown } x$$

8. Applying the Gauss elimination method, solve the following systems of equations

$$\begin{cases} x + 2y + z = 3 \\ 3x + 2y + z = 3 \\ x - 2y - 5z = 1 \end{cases}$$

$$\begin{cases} x + 2y + 4z - 3t = 0 \\ 3x + 5y + 6z - 4t = 1 \\ 4x + 5y - 2z + 3t = 1 \end{cases}$$

9. Applying the Kronecker-Capelli theorem, show that the system

$$\begin{cases} x + 2y + 3z - t = -1 \\ 3x + 6y + 7z + t = 5 \\ 2x + 4y + 7z - 4t = -6 \end{cases}$$

has infinitely many solutions and then solve this system.

10. Applying the Kronecker-Capelli theorem, show that the system

$$\begin{cases} x - y - 2z + 2t = -2 \\ 5x - 3y - z + t = 3 \\ 2x + y - z + t = 1 \\ 3x - 2y + 2z - 2t = -4 \end{cases}$$

is inconsistent.

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