# Partial differential equations
Project instructions and rules

## 1 Important dates

- Choosing a topic: **2 April**.
- Project deadline: **9 June**.

## 2 Rules

### 2.1 Choosing a topic and returning the project

First, you will have some time to form a group and choose a topic until the given date (see above).

- Project groups have to consist of **2 persons**, but in certain cases a 3 person groups can be formed.
- Each group will choose two topics: primary and secondary. I will try to assign the first choice for everybody but in a case of over-repetition the second choice will be given.
- Your group composition and chosen topics should be to me until the given date. **Only one** person from the group sends me the message with a title

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PDE - project assignment
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- As soon as possible I will publish the assigned topics on my webpage.
- Use **Jupyter**.

At the end of the semester there will be time to return the project.

- ZIP all your files and make sure that everything needed to execute it is attached.
- Upload your file to **ePortal** in a respective folder.

### 2.2 Project contents

When the work begins please focus on several important points.

- Implementation. You can code in any language you want (well, almost). However, the optimal choice will be to use a Notebook-admissible one. Using Jupyter is **obligatory**. It will let you place your code and comments into one document which will greatly facilitate my work afterwards.
- Documentation. Your notes accompanying the code should contain some theoretical explanation of the method used such as derivation, nomenclature and some simple proofs where possible.

The overall aim of the project is to **learn about numerical methods for PDEs in a practical way by implementing them and verifying some theoretical results**. In particular, you should focus on the following items.

- Consistency. When you choose a finite difference method you have to prove its consistency. It usually is done during the derivation of the method.
- Convergence. If a method converges, it does so at a particular pace. The numerical task is to check it by a simulation. This is easy if you know the exact solution: compute the error (for example absolute) for different discretization parameters (for example the grid spacing) and plot it on a log-log scale. The graph should resemble a line which tangent is the order of the method (why?).
- Stability. Some methods are only conditionally stable. If these given conditions are not met, the round-off error can be exponentially magnified. It is a good idea to explore this numerically.

**All relevant definitions and theorems** (if you use any) should be stated in the documentation. If you choose a finite element method you will have to clearly construct it for a given equation by writing it in a weak form, and choose approximation spaces with corresponding bases.

Everything else is up to your creativity. If you would like to write a GUI, please do it. If you would like to generate some animations, your are welcome. If you would like to play with some data, have fun with that.

## 3 Topics

You can choose **any topic** concerning numerical methods for PDEs you would like to. You can also use the following *topic generating algorithm*.

1. Choose an equation type.
   - First order linear PDEs (no shock waves).
   - First order nonlinear PDEs (with shock waves).
   - Parabolic PDEs (e.g. heat equation).
   - Elliptic PDEs (e.g. Laplace's and Poisson's equation).
   - Hyperbolic PDEs (e.g. wave equation).
2. Choose a method.
   - Finite difference method.
   - Finite element method.
   - Meshless method.
   - Spectral method.
   - ...

Taking the above two steps will leave you with a nice topic to explore. Keep in mind that some combinations can be easier than others and thus require more detailed elaboration. In other words, choosing a more difficult topic will put some more emphasis on understanding the theory. Of course, I will take this into account during grading.

**Referring to applications is also a very good idea!**

## 4 Literature

The best place to start is to read some literature. Below is a collection of some very good texts.


1. **Leveque**, *Finite Difference Methods for Ordinary and Partial Differential Equations*. This is a very good introductory book on various finite difference methods. In particular, it nicely covers Elliptic, Parabolic and Systems of Hyperbolic equations.
2. **Leveque**, *Numerical Methods for Conservation Laws*. The second volume of the above. Here, you will find methods for conservation laws with shock waves.
3. **Brenner, Scott**, *The Mathematical Theory of Finite Element Methods*. One of the best introductory books on Finite Element Method. It contains everything (and more) you want to know about them.
4. **Griffiths**, *Numerical Analysis Using R: Solutions to ODEs and PDEs*. A very good application-oriented book. It contains complete codes in R and covers a wide scope of schemes: finite differences, methods for conservation laws, MOL, and meshless methods.


*Łukasz Płociniczak*