Wandering Seminar
Ergodic Theory & Dynamical Systems
Wroclaw University of Technology
Wrocław, 2–5 March 2017
http://prac.im.pwr.wroc.pl/~wandering/

Abstracts of talks

February 27, 2017

Mini-course, Friday–Sunday, March 3–5

Symbolic Dynamics and Factor Maps

Aimee Johnson (Swarthmore College)

Symbolic dynamical systems developed as a useful tool in studying general dynamical systems and as a model of data storage for computers. In this context, the symbolic models consist of one-dimensional arrays of symbols and a shift map that maps one such array to another. One can ask the same questions of these systems as one asks for any dynamical system: for instance, when are two such systems conjugate? When can we map from one system to another? One wonderful result in this vein says that if the entropy of the symbolic dynamical system is at least log(n), then one can find a map from the system to the full shift on n symbols. We will discuss this result and consider the analogous situation for higher dimensional symbolic systems. We will follow the story of what happens for this situation, delving into the details and proofs as much as possible.

Lectures:

1. Introduction to higher dimensional symbolic dynamical systems.
2. Factoring onto the full shift when we have an extra mixing condition and enough entropy: the work of Johnson and Madden.
3. The complete story for factoring onto the full shift in the above situation: the work of Desai.
4. The equal entropy situation: the work of Boyle and Schraudner.
5. What can happen without the mixing condition: the work of Pavlov.

Introductory lecture, Thursday, March 2

Dawid Huczek

The goal of the talk is to provide prerequisites for the minicourse presented by Aimee Johnson. The following topics will be covered:

- One-dimensional subshifts (basic definitions, words and cylinders, forbidden blocks, language, sliding block codes and higher block codes)
- Entropy in symbolic systems
- Topological mixing

Talks (Friday–Sunday, March 3–5) in alphabetical order

The torsion problem for the automorphism group of a Z^d-full shift and its topological fullgroups.

Sebastian Barbieri

We introduce the group $RTM(G, n, k)$ composed of abstract Turing machines which use the group G as a tape, use an alphabet of n symbols, k states and act as a bijection on the set of configurations. These objects can be represented both as cellular automata and in terms of continous functions and cocycles. The study of this group structure yields interesting results concerning computability properties of some well studied groups such as $\mathrm{Aut}(A^{\mathbb{Z}})$ and the topological full group of the two dimensional full shift.
More precisely, given a finitely generated group $G = \langle S \rangle$, we define its torsion problem as the language formed by the words $w \in S^*$ which represent torsion elements in G. We show that the two examples mentioned above contain finitely generated subgroups for which the torsion problem is undecidable.
These results are from joint work with Jarkko Kari and Ville Salo.

Automorphism group of Toeplitz $\mathscr{B}$-free systems

Aurelia Bartnicka

For a given subset $\mathscr{B} \subset \mathbb{N}$, we are interested in the associated set of $\mathscr{B}$-free numbers $\mathcal{F}_{\mathscr{B}} := \mathbb{Z} \setminus \bigcup_{b \in \mathscr{B}} b\mathbb{Z}$. We consider the orbit closure of $\eta := \mathbb{1}_{\mathcal{F}_{\mathscr{B}}} \in \{0,1\}^{\mathbb{Z}}$ under the left shift S. Many natural examples of $\mathscr{B}$-free shifts are not minimal, for example, in the square-free case the corresponding subshift is proximal and has positive entropy. We will show that a $\mathscr{B}$-free shift is minimal if and only if it is a Toeplitz subshift. Moreover, for a certain class of Toeplitz $\mathscr{B}$-free subshifts we will prove that their automorphism groups are trivial.

Sparse generalised polynomials and nilmanifolds.

Jakub Byszewski

We investigate generalised polynomials (i.e. polynomial-like expressions involving the use of the floor function) which take the value 0 on all integers except for a set of density 0. By a theorem of Bergelson-Leibman, generalised polynomials can be completely described in terms of dynamics on nilmanifolds. Our main result is that the set of integers where a sparse generalised polynomial takes non-zero value cannot contain a translate of an IP set. We also study some explicit constructions, and show that the characteristic functions of the Fibonacci and Tribonacci numbers are given by generalised polynomials. Finally, we show that any sufficiently sparse 0,1-valued sequence is given by a generalised polynomial. We apply these results to a question on automatic sequences.

Period-like motions in flows

Karol Gryszka

Almost a century ago Birkhoff and Bohr introduced their notions of recurrence and almost periodicity. These two notions remain ones of the best known and most useful generalizations of periodic motions.
Since then many variations of period-like motions have been introduced. Most of them are well-described and also characterized using other period-like motions and/or stability properties. That rises to one of the most impotant question: how do they interact with each other? In the general case there are very few relations. The goal of the talk is to provide a diagram of such relations.

The number of ergodic invariant measures for finite rank Bratteli diagrams

Olena Karpel

The main goal of this talk is to give an explicit description of the set of all invariant probability measures on a Bratteli diagram $B = (V, E)$ of finite rank k. This set is a simplex $\Delta(B)$ with l vertices, where l is some number between 1 and k. The vertices of $\Delta(B)$ correspond to the ergodic invariant probability measures on B. We determine the vertices of $\Delta(B)$ in terms of the incidence matrices of B.
This is a joint work with S. Bezuglyi and J. Kwiatkowski.

On minimal homeomorphisms preserving foliations

Wojciech Kozłowski

We apply the decomposition theory of a manifold elaborated by Bing and Daverman, to study of a decomposition space of a foliated manifold. We consider a minimal homeomorphism preserving foliation on a compact manifold. As a consequence of our main result we obtain a generalization of the result of Kolyada, Snoha and Trofimchuk concerning a minimal skew-product homeomorphism of the 2-torus.
The talk is based on the joint paper with Andrzej Biś.

The Weyl Pseudometric for an Amenable Group Actions

Martha Łącka

During the talk we will present some consequences of the convergence with respect to the Weyl pseudometric in dynamical systems generated by an amenable group actions. This will lead us to an alternative proof of the Krieger theorem, which says that for any number between 0 and $\log k$ one can find a Toeplitz shift over a k-letter alphabet with entropy equal to this number.
The talk will be based on a joined work with Marta Pietrzyk.

Some invariants for von Neumann special flows

Anton Solomko

A von Neumann flow is a special flow over an irrational rotation of the circle and under a piecewise smooth roof function with a non-zero sum of jumps. Such flows appear naturally as special representations of Hamiltonian flows on the torus with critical points. We consider the class of von Neumann flows with one discontinuity. I will show that any such flow has infinite rank and that the absolute value of the jump of the roof function is a measure theoretic invariant, that is two ergodic von Neumann flows with one discontinuity are not isomorphic if the jumps of the roof functions have different absolute values, regardless of the irrational rotation in the base. The main ingredient in the proofs is a Ratner type property of parabolic divergence of orbits of two nearby points in the flow direction.
Joint work with Adam Kanigowski.

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