Optimization Theory

Applied Mathematics

Problem set 4

1. ([AEP07]) Formulate the following problem as an integer linear programming problem:

   A large chain of department stores wants to build a number of distribution centers (warehouses) which will supply 30 department stores with goods. They have 10 possible locations to choose between. To build a warehouse at location i ($i = 1, \dots, 10$) costs c_i and the capacity of a warehouse at that location would be k_i volume units per week. Department store j has a demand of e_j volume units per week. The distance between warehouse i and department store j is d_{ij} km, $i = 1, \dots, 10$, $j = 1, \dots, 30$, and a certain warehouse can only serve a department store if the distance is at most D km. The investors' goal is to minimize the cost of investing in the necessary distribution centers.

2. At the beginning of a day the manager of a hotel for hours obtains a list of potential customers with the (positive integer) numbers of hours they want to spend in their hotel t_i, $i = 1, \dots, 100$. The manager has to assign each person to at most one of the 20 identical rooms in such a way that everyone served leaves within 23 hours and no two customers stay in one room at the same time. Moreover, after each customer's stay the room is taken over by the room service for an hour, after which another customer can enter the room. The goal of the manager is to maximize the profit of the hotel computed as the sum of the payments by all the served customers minus the cost of the room service. A payment each customer makes consists of a \$15 entrance fee and \$10 per each hour of usage. The cost of the room service depends on the hour the last customer leaves the hotel T and is equal to \$120 times $T + 1$.

   Write (without solving) an integer program allowing you to find the optimal assignment of guests to rooms. Write also its matrix form (with constraints in the form of inequalities).

3. A small factory works 5 days per week and produces two products: product A and product B. The weekly production has to satisfy the demand of 100 items of product A and 50 items of product B.

   The factory has two types of employees: dwarfs, who assemble parts of the products with their tiny hands, and giants, who use their power to produce the most complicated parts from stainless steel. Dwarfs work only on odd days of week and giants work only on even days. There are 30 dawrfs and 25 giants available for work. If an employee is hired, he works an 8-hour shift on each of the days for which he is hired (e.g. each hired giant works two 8-hour shifts – one on tuesday and one on thursday and each dwarf works three 8-hour shifts – one on monday, one on wednesday and one on friday). Each item of product A needs 1 hour of assembling, then 2 hours of producing additional parts and then 2 more hours of assembling. Moreover, we assume that once the assembling/part producing work is started, it cannot be postponed to the next day (that is – to produce one item of product A, work has to be done on exactly three different days). Similarly, an item of product B needs 3 hours of part-producing and then 1 hour of assembling (which, again, need to be done on exactly two different days).

   Write (without solving) an integer program which allows the owner of the factory to design the production plan for the two goods, which minimizes the total number of employees hired.

4. An energy plant consists of 3 blocks: block A with peak energy production of 8GW per hour, block B with peak energy production of 6GW per hour and block C with peak energy production of 5GW per hour. The manager of the plant must decide which blocks should operate at which months of the year in order to cover the demand in each month while minimizing the energy production cost computed as the sum of energy costs of all operating blocks over all 12 months. The average hourly demands for each month in gigawatts are given in the table below:

month	1	2	3	4	5	6	7	8	9	10	11	12
demand	16	17	12	11	9	7	8	6	10	11	12	13

   The energy production cost and the hourly production of each block depend on whether it is within its one-month starting period (after a period of staying idle) or it is in the period of regular production. In the starting period the energy production is only a half of its peak production, while in the normal period it is equal to the peak production. As for the cost, for each block it is \$9000 per hour in the starting period and \$6500 per hour in the normal period (hence, it does not make sense to restart a block many times within each year).

Write (without solving) an integer program allowing you to find out which blocks should be operating in which months so that the cost is minimized subject to all the demand constraints.

5. Suppose identical packages of covid19 vaccines are supposed to be delivered to 57 locations around Poland from the headquarters of the state agency responsible for their distribution. The deliveries are made using just 1 car. The distances in kilometers d_{ij} between different locations $i, j = 1, \ldots, 57$ as well as the distances between the headquarters and each location $d_{0i} = d_{i0}$, $i = 1, \ldots, 57$ are known. Suppose car can visit locations in any order provided that after (at most) each 2000 kilometers (including the return route) it returns to the base for the car check-up and refuelling.

   The goal of the manager in the agency is to determine the ordering in which the different locations should be visited by the car in such a way that each location (except the headquarters) is visited exactly once and that the number of returns to the headquarters is the smallest possible. You can use the fact that in case the locations are visited in alphabetical order the number of returns is 10. The problem can be solved using an integer program.

   Formulate (without solving) this program.

6. Solve the integer program

$$\begin{array}{ll} \text{maximize} & 2x_1 + x_2 \\ \text{subject to} & 3x_1 + x_2 \leq 10, \\ & 2x_1 + 6x_2 \leq 27, \\ & x_1 \geq 0, x_2 \geq 0, x_1, x_2 \in \mathbb{Z} \end{array}$$

   using the branch and bound procedure. Solve each linear-programming problem encountered graphically.

7. Solve the following integer program:

$$\begin{array}{ll} \text{maximize} & 4x_1 + 3x_2, \\ \text{subject to:} & x_1 + 3x_2 \leq 12, \\ & 3x_1 + 2x_2 \leq 21, \\ & -2x_1 + 6x_2 \leq 7, \\ & x_1 \geq 0, \ x_2 \geq 0 \text{ and integer} \end{array}$$

   using the branch and bound procedure. Solve each linear-programming problem encountered graphically.

References:

[AEP07] N. Andreasson, A. Evgrafov, M. Patriksson, An Introduction to Continuous Optimization, Studentlitteratur AB, 2007