Optimization Theory

Applied Mathematics

Laboratory assignments

LP1. (Based on [MK80]) The Department of Agriculture is encouraging better crop planning. In order to acheive that it rents some land to the farmers from three provinces A, B and C under the condition that the amounts of different crops planted are such that the total revenue is maximized. These amounts should differ according to natural conditions in each of the regions as presented in the table below

Region	For rent [acres]	Water availability [gallons]	Percentage of 1st class grounds
A	8400	9500000	60
B	9700	20000000	70
C	4500	750000	45

Crops data is as follows:

Crop	Water consumption [gallons/acre]	Is 1st class soil necessary	Profit [$/acre]
Millet	1200	No	150
Cane	3000	No	470
Cotton	2100	Yes	390
Patatoes	1500	No	260

What should be planted in what proportion in each region if the goal is to maximize the profit subject to the following constraints:

(a) On at least 7500 acres (in all regions together) patatoes should de planted.

(b) The crops must satisfy all the constraints due to their water consumption and the class of soil necessary for their cultivation.

Write and solve the linear program allowing you to find the optimal plan for the Agriculture Department.

LP2. Knoxville has to get rid of 7500 tons of trash every day. It burns as much of them as possible in 4 incinerators. The data concerning them is given in the table below:

Incinerator	Daily capacity [t]	Emission of SO_2 per ton	Emission of particulate per ton
1	900	250	22
2	2000	230	20
3	1400	130	30
4	1200	220	25

The cost of using the incinerators is $16 per each ton of garbage burnt. The state Environmental Quality Commision has determined the limits on emissons of SO_2 and particulate, which are 500000 for SO_2 and 60000 for particulate. If some of the garbage is not burnt in any of the incinerators, it must be transported to one of two land fills. A small land fill, where at most 1000 tons of trash can be transported, is situated nearby. Puting each ton of garbage there costs $23. The second land fill is in the neighbouring state, and it costs $57 per each ton. The town has to determine, how much garbage can be burnt in each incinerator and how much has to be sent to each land fill, so that the norms are satisfied and the total cost is minimized. Create and solve the linear program allowing you to find it out.

LP3. A company producing a single product has 2 production centers A and B, 3 warehouses C, D and E and 12 retail outlets. All the production of the company is first transported from one of the production centers to a warehouse and next from a warehouse to outlets. Both production centers can produce as much product as necessary. Daily capacity of each warehouse is 2500 units of the product, while daily demands for the product are different for each outlet and given in the table below:

1	2	3	4	5	6	7	8	9	10	11	12
520	250	220	100	1500	250	980	210	360	290	750	660

A mathematician has been hired to decide what amount of product should be produced in each production center and what amounts should next be transported from each production center to each warehouse and from each warehouse to each outlet. The plan he is supposed to create should minimize the overall cost of

transportation taking into account that the cost of transportation between any production center and any warehouse is \$0.09 per mile per unit, while the cost of transportation between any warehouse and any outlet is \$0.24 per mile per unit. The distances (in miles) between production centers and warehouses are:

	C	D	E
A	35	67	178
B	109	82	25

while those between warehouses and retail outlets are:

	1	2	3	4	5	6	7	8	9	10	11	12
C	25	16	150	281	93	25	229	312	111	110	40	75
D	40	59	102	170	100	40	210	281	160	203	71	59
E	77	87	69	265	15	78	150	243	76	145	130	101

Write and solve the linear program allowing to find the optimal production and transportation plan for the company.

LP4. (Based on [CoTu06]) A company will face the following cash requirements in the next eight quarters (positive entries represent cash needs while negative entries represent cash surpluses):

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
600	250	100	−500	400	−900	350	−700

The company has four borrowing possibilities:

- A 2-year loan available at the beginning of Q1, with a 7.0% interest per quarter.
- A 1-year loan available at the beginning of each year (i.e. at the beginning of Q1 and Q5), with a 9.4% interest per quarter.
- The two remaining borrowing opportunities are available at the beginning of every quarter: a 6-month loan with a 11.6% interest per quarter, and a quarterly loan with a 13.5% interest for the quarter.

Any surplus can be invested at a 4.5% interest per quarter. Formulate and solve a linear program that maximizes the wealth of the company at the beginning of Q9.

LP5. Consider a company making a single product. The demand for the next four days is 90, 200, 150 and 140 units. The company employs at most 15 people and each person works for 8 hours daily. Each person can make one unit of the product per hour. The employees can work for 2 extra hour per day overtime. The company uses different systems to pay for regular and overtime work. The regular work is paid \$400 per day of work, regardless of the amount of product made by the person (i.e. regardless of whether they have worked for the entire day or stayed idle for several hours. Cost of over time production is \$70 per product. The company has to meet the daily demand but can produce more and store them for further use at \$10 per unit per day. The company can get the product made outside and buy it at \$125 per unit. Find the number of people to be employed and the least cost production plan for those people.

LP6. (Based on [NPTEL14]) Consider a caterer who has to provide food for several dinners happening in the next 8 days. The demand for cloth napkins that are used in the dinners is 89, 130, 109, 100, 70, 150, 206 and 151. New napkin costs \$7.5. Napkins can be put to laundry and washed napkins can be used on subsequent days. Two types of laundry are available. The fast laundry that charges \$2.9 per unit and will deliver for use on the second day and the slow laundry that costs \$1.3 per unit and can deliver for use on the third day. Find the least cost purchase and use plan for the caterer using linear programming (the optimal solution should be integer without adding integrity constraint).

LP7. Suppose you want to construct a portfolio consisting of actions of five companies: Heineken, Phillips, Getronicks, Unilever and Fortis, which will minimize the overall portfolio's mean absolute deviation while giving at

least 3.5% of expected monthly profit. The data about each company's returns is given in the table below:

	Heineken	Phillips	Getronicks	Unilever	Fortis
2000	1514.33	516.08	124.17	168.45	297.16
2001	1706.15	547.29	151.19	187.38	279.90
2002	1916.69	512.93	167.15	175.14	305.14
2003	1879.16	483.79	144.17	206.69	280.10
2004	2223.23	569.11	195.62	229.20	305.81
2005	2615.33	579.14	273.18	278.11	287.61
2006	2709.10	587.25	215.91	297.51	319.59
2007	3154.16	691.05	244.69	345.11	374.65
2008	2499.18	534.81	218.77	269.13	353.58
2009	2322.58	573.01	288.29	287.50	335.30
2010	2489.01	641.12	275.13	321.21	384.97
2011	2611.97	690.16	312.93	379.11	454.03
2012	3475.15	669.33	317.88	396.14	496.00
2013	3789.20	726.11	389.12	368.76	526.09
2014	3415.75	758.36	374.51	341.51	501.51
2015	3819.00	800.90	409.16	302.05	534.64
2016	3725.16	801.43	451.97	297.11	536.70
2017	3985.45	753.16	444.25	347.85	545.91
2018	4216.81	802.06	509.21	376.19	544.36
2019	4590.72	874.25	517.61	409.23	588.25
2020	3711.67	842.11	411.45	325.19	508.31
2021	3866.11	811.99	463.15	336.78	509.24
2022	4067.59	866.51	506.50	356.10	560.28

For the information about how a mean absolute deviation portfolio is constructed see e.g. [CoTu06], Chapter 8. To see how the returns are computed from the data given in the table, see [DX15].

LP8. (Based on [MK80]) The American Safety Council has allocated $250000 to efforts to prevent automobile accidents. Two assumed measures of effectiveness of such efforts are the reduction in fatalities and in property damage. The projects suggested and relevant expected reductions per $1000 invested are given in the table below:

Project	Exp. fatality reduction	Exp. prop. dam. reduction
Education in schools	18	$10
Education for perpetrators	12	$35
Promotion of seat-belt usage	20	$0
Promotion of not using mobiles while driving	18	$70
Additional financing of traffic police	24	$15
Research in improved vehicle design	3	$16

The main goal of these projects is the reduction of fatalities, hence, the council seeks for the allocation of funds which maximizes the expected reduction of fatalities, subject to the following constraints:

(a) The expected reduction in property damage should be at least $5000.

(b) At least half of the funds are invested in educational or scientific programs.

(c) No more than 30% of the funds can be given to the public sector (i.e. schools or the police).

Write an appropriate linear program allowing to find the optimal allocation of funds.

References:

[CoTu06] G. Cornuejols, R. Tütüncü, Optimization Methods in Finance, Cambridge University Press, 2007

[DX15] Z. Donovan, M. Xu, Quadratic Programming: Applications (lecture notes), available online at http://community.wvu.edu/ ∼krsubramani/courses/sp15/optfin/lecnotes/QPapps.pdf

[MK80] C.K. McKenna, Quantitative Methods for Public Decision Making, 1st Ed., McGraw-Hill College, 1980

[NPTEL14] Online course Operations Research Applications – Linear and Integer Programming. Available at https://nptel.ac.in/courses/110/106/110106059