Theory and Methods of Optimization

Embedded Robotics

Problem set 2

1. ([AEP07]) Formulate the following problem as an integer programming problem. A large chain of department stores wants to build a number of distribution centers (warehouses) which will supply 30 department stores with goods. They have 10 possible locations to choose between. To build a warehouse at location i ($i = 1, \ldots, 10$) costs c_i and the capacity of a warehouse at that location would be k_i volume units per week. Department store j has a demand of e_j volume units per week. The distance between warehouse i and department store j is d_{ij} km, $i = 1, \ldots, 10$, $j = 1, \ldots, 30$, and a certain warehouse can only serve a department store if the distance is at most D km. The investors' goal is to minimize the cost of investing in the necessary distribution centers.

2. n tasks are to be assigned to 1 of 4 identical processors P_1, P_2, P_3 and P_4. Performing task i on any processor takes $t_i > 0$ nanoseconds. A schedule for the processors is given by four disjoint subsets S_1, S_2, S_3 and S_4 of $\{1, \ldots, n\}$ such that $S_1 \cup S_2 \cup S_3 \cup S_4 = \{1, \ldots, n\}$. Tasks from set S_i are performed on processor P_i in an arbitrary order. The time when all the tasks are completed can be computed as

$$T_{\max} = \max \left\{ \sum_{i \in S_1} t_i, \sum_{i \in S_2} t_i, \sum_{i \in S_3} t_i, \sum_{i \in S_4} t_i \right\}.$$

   Write an integer program finding a schedule minimizing $T_{\max}$.

3. The demand for a single product for the upcoming week is given in the table below.

Day	Mon	Tue	Wed	Thu	Fri
Demand	37	25	50	42	30

   Its producer has 3 machines on which he can produce it: at most 20 units may be produced daily on machine A, at most 20 on machine B and at most 25 on machine C. The cost of production on each machine consists of two parts: fixed cost of \$2000 which has to be paid if the machine is used (for any amount of time) over the course of the week and running cost proportional to the number of units of product made on a given machine equal to \$15 per unit for machine A, \$20 per unit for machine B and \$25 per unit for machine C. The demand for each day must be met, but the production may be done on a previous day and kept in a storage. The cost of storing a unit of the product overnight is \$5.

   Write (without solving) the mixed integer program allowing to find the optimal production plan.

4. An industrial machine consists of 3 units: A, B and C. Each unit can be produced in several ways out of sub-assemblies of 5 types (type 1, type 2, etc.) The way to assemble each unit from sub-assemblies of types 1 and 2 is given in the table below:

Unit	Type 1 sub-assemblies	Type 2 sub-assemblies
A	2	2
B	1	4
C	3	5

   The functionality of type 1 sub-assembly can be obtained by constructing its equivalent from 2 sub-assemblies of type 4 and 1 of type 5. That of type 2 sub-assembly from 3 sub-assemblies of type 3, 1 of type 4 and 1 of type 5. The costs of each type of sub-assembly and their energy consumption when they work are given in the table below:

	Type 1	Type 2	Type 3	Type 4	Type 5
Cost [\$]	16	25	3	5	2
Energy consumption [W]	52	83	24	17	32

   Which parts should be used to produce each unit if the goal of the designer is to minimize the cost subject to the total energy consumption being not bigger than E. Write (without solving) the integer programming problem which answers this question.

5. Solve the following problem

$$\begin{array}{ll} \text{maximize} & x_1 + x_2 \\ \text{subject to} & 2x_1 + 6x_2 \leq 31, \\ & 2x_1 + x_2 \leq 9 \\ & x_1 \geq 0, x_2 \geq 0 \text{ and integer} \end{array}$$

   using the Branch and Bound procedure. Solve each LP subproblem graphically.

6. Solve the following integer program:

$$\begin{array}{ll} \text{maximize} & x_1 + 2x_2, \\ \text{subject to:} & 2x_1 + 6x_2 \leq 27, \\ & 2x_1 + x_2 \leq 12, \\ & -x_1 + 3x_2 \leq 5, \\ & x_1 \geq 0,\ x_2 \geq 0 \text{ and integer} \end{array}$$

using the branch and bound procedure. Solve each linear-programming problem encountered graphically.

References:

[AEP07] N. Andreasson, A. Evgrafov, M. Patriksson, An Introduction to Continuous Optimization, Studentlitteratur AB, 2007,