# Theory and Methods of Optimization 

Embedded Robotics

Computer assignments
LP1. Murderville has specified the minimum requirements for the number of patrolmen on duty during each 4-hour period as seen in the table below:

| Time of day | Number of patrolmen |
| :---: | :---: |
| Midnight-4A.M. | 36 |
| 4A.M.-8A.M | 18 |
| 8A.M.-Noon | 12 |
| Noon-4P.M. | 10 |
| 4P.M.-8P.M | 20 |
| 8P.M.-Midnight | 32 |

Each policeman works for consecutive 8 hours during the day, and his assignment is repeated on each day (in particular, he may be assigned to a shift starting on one day and ending on the next one). Moreover, at least $30 \%$ of policemen working at any time must be officers with at least 4 -year experience in the force. The total number of such officers available is 23 . The number of less experienced officers can be considered unlimited. Find the assignment of experienced and unexperienced officers to each of the 8-hour shifts which minimizes the total number of policemen used and satisfies all the requirements imposed by the town. Solve the problem using a linear programming model.

LP2. The Education Council in a town in the mid-west of the United States tries to plan the assignment of students from 3 neighbourhoods to the 2 public schools in town. The assignment is supposed to minimize the total cost of school busing subject to the constraints related to the number of students each school can accomodate as well as requirements of the racial balance policy. The data realated to the problem is given in the table below (costs are given in dollars per person):

| Neighbourhood | No. of black students | No. of white students | Cost of bus to school 1 | Cost of bus to school 2 |
| :--- | :---: | :---: | :---: | :---: |
| A | 1100 | 0 | 0 | 13 |
| B | 720 | 960 | 15 | 10 |
| C | 510 | 760 | 17 | 5 |

The capacity of school 1 is 2500 students, while the capacity of school 2 is 2200 students. The racial balance policy requires that there are no more than $70 \%$ students of each race in each school.
Formulate and solve the integer programming model that will find the optimal assignment of students to schools satisfying all the constraints.
LP3. Suppose five daily newspapers are published in a certain country, each paper covering some of the twelve regions of the country as shown in the following table:

| Newspaper | Regions covered | Cost of advertisment | Expected benefit |
| :---: | :---: | :---: | :---: |
| 1 | $1,2,4,9$ | $\$ 3000$ | $\$ 12000$ |
| 2 | $2,3,6,8,10,12$ | $\$ 7000$ | $\$ 14000$ |
| 3 | $4,5,7,10$ | $\$ 4000$ | $\$ 10000$ |
| 4 | $2,3,8,10,11$ | $\$ 5000$ | $\$ 19000$ |
| 5 | $4,5,6,9,11,12$ | $\$ 8000$ | $\$ 20000$ |

The marketing manager wants to find an advertisment maximizing the expected total profit such that
(a) The advertisment covers the whole country.
(b) The sum of total costs of advertisment will not exceed 20000 and will be at most one third of the expected profit from the advertisment.
Formulate the manager's problem as an integer linear program and solve it.
LP4. Suppose a restaurant working 7 days a week wants to hire waiters. Based on past experience, the number of waiters needed on each day of the week can be given as follows:

| Day | Mon | Tue | Wed | Thu | Fri | Sat | Sun |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Demand | 11 | 9 | 15 | 13 | 17 | 19 | 22 |

There are three types of waiters: full-time waiters work for 5 consecutive days and then rest for the next two; part-time waiters work for 3 consecutive days and then rest for the next 4 days, week-end waiters work only on saturday and sunday. Each worker repeats his weekly working pattern indefinitely. Full-time waiters are paid $\$ 14$ per day of work, part-time waiters are paid $\$ 16$ per day of work, while week-end waiters are paid $\$ 12$ per day of work. Provide the waiters' work-plan for the week minimizing the cost for the restaurant. The plan must satisfy all the demand constraints. Do it using an integer programming model.

LP5. A supermarket working 7 days a week wants to hire employees. The number of people needed on each day of the week are known to be:

| Day | Mon | Tue | Wed | Thu | Fri | Sat | Sun |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Demand | 23 | 19 | 24 | 17 | 25 | 39 | 30 |

There are two types of candidates for this work: full-time employees work for 5 consecutive days and then rest for the next two, they can start their shift on any day of the week and are paid $\$ 16$ per day of work; part-time employees work only on weekends (that is - either on friday and saturday or on saturday and sunday) and are paid $\$ 13$ per day of work. Each worker repeats his weekly working pattern indefinitely. Provide the supermarket's work-plan for the week minimizing the cost of labour. The plan must cover all the demand on every day of the week. Do it using an integer programming model.

LP6. (Based on [MK80]) The American Safety Council has allocated $\$ 250000$ to efforts to prevent automobile accidents. Two assumed measures of effectiveness of such efforts are the reduction in fatalities and in property damage. The projects suggested and relevant expected reductions per $\$ 1000$ invested are given in the table below:

| Project | Exp. fatality reduction | Exp. prop. dam. reduction |
| :---: | :---: | :---: |
| Education in schools | 18 | $\$ 10$ |
| Education for perpetrators | 12 | $\$ 35$ |
| Promotion of seat-belt usage | 20 | $\$ 0$ |
| Promotion of not using mobiles while driving | 18 | $\$ 70$ |
| Additional financing of traffic police | 24 | $\$ 15$ |
| Research in improved vehicle design | 3 | $\$ 16$ |

The main goal of these projects is the reduction of fatalities, hence, the council seeks for the allocation of funds which maximizes the expected reduction of fatalities, subject to the following constraints:
(a) The expected reduction in property damage should be at least $\$ 5000$.
(b) At least half of the funds are invested in educational or scientific programs.

Write an appropriate linear program allowing to find the optimal allocation of funds.
LP7. (Based on [MK80]) The Department of Agriculture is encouraging better crop planning. In order to acheive that it rents some land to the farmers from three provinces $\mathrm{A}, \mathrm{B}$ and C under the condition that the amounts of different crops planted are such that the total revenue is maximized. These amounts should differ according to natural conditions in each of the regions as presented in the table below

| Region | For rent [acres] | Water availability [gallons] | Percentage of 1st class grounds |
| :---: | :---: | :---: | :---: |
| A | 8400 | 9500000 | 60 |
| B | 9700 | 20000000 | 70 |
| C | 4500 | 750000 | 45 |

Crops data is as follows:

| Crop | Water consumption [gallons/acre] | Is 1st class soil necessary | Profit [\$/acre] |
| :---: | :---: | :---: | :---: |
| Millet | 1200 | No | 150 |
| Cane | 3000 | No | 470 |
| Cotton | 2100 | Yes | 390 |
| Patatoes | 1500 | No | 260 |

What should be planted in what proportion in each region if the goal is to maximize the profit subject to the following constraints:
(a) On at least 7500 acres (in all regions together) patatoes should de planted.
(b) The crops must satisfy all the constraints due to their water consumption and the class of soil necessary for their cultivation.

Write and solve the linear program allowing you to find the optimal plan for the Agriculture Department.
LP8. Knoxville burns 5000 tons of trash every day in 4 incinerators. The data concerning them is given in the table below:

| Incinerator | Daily capacity $[\mathrm{t}]$ | Emission of $S O_{2}$ per ton | Emission of particulate per ton |
| :---: | :---: | :---: | :---: |
| 1 | 900 | 250 | 22 |
| 2 | 1700 | 230 | 20 |
| 3 | 1400 | 130 | 30 |
| 4 | 1000 | 220 | 25 |

The state Environmental Quality Commision has determined the limits on emissons of $\mathrm{SO}_{2}$ and particulate, which are 500000 for $S_{2}$ and 60000 for particulate. The town has to determine, how much garbage can be burnt in each incinerator so that the norms are satisfied (the rest must be transported to land fill in the neighbouring state, which is a much more expensive way to get rid of it). Create the linear program allowing you to find it out.

LP9. The increase of the cost of coal has forced the Knoxville Municipal Power Company to seek some savings in their energy production plan. The company has to produce 8000 MWh of energy every day subject to the state norms of emissions of particulate and $\mathrm{SO}_{2}$. The cost of producing $1 M W h$ of energy in $\$$ and the levels of emissions of particulate and $\mathrm{SO}_{2}$ in $\mu \mathrm{g}$ per 1 MWh of energy are as follows:

| Coal | Production cost | Emision of particulate | Emission of $\mathrm{SO}_{2}$ |
| :---: | :---: | :---: | :---: |
| Colombian | 42 | 3.5 | 80 |
| Indonesian | 18 | 12 | 270 |

The daily norm for the emission of particulate is $60000 \mu \mathrm{~g}$, while that for the amission of $\mathrm{SO}_{2}$ is $300000 \mu \mathrm{~g}$.
There's also a possibility of installation of filters on the chimneys of the plant. In that case the cost of producing $1 M W h$ of energy increases by $\$ 20$ (for each type of coal), but the emission of particulate decreases by $90 \%$, while that of $\mathrm{SO}_{2}$, by $80 \%$. Some amount of energy can also be bought at the market price of $\$ 57$ per 1 MWh . Write an integer program allowing you to find out, how much Colombian and how much Indonesian coal should be used for production, as well as whether the filters should be installed or not and whether some fraction of energy should be bought at the market. Your goal is the minimization of the total cost.

LP10. The head of the building department is planning the inspections for the upcoming week. He has 3 inspectors at his disposal: plumbing inspector ( 28 h per week for inspections), electrical inspector ( 30 h for inspections) and building inspector ( 34 h for inspections). The times necessary for inspecting different types of buildings are given in the table below:

| Inspector | Gas station | Restaurant | Garage | Residential building |
| :---: | :---: | :---: | :---: | :---: |
| plumbing insp. | 4 | 2 | 1 | 2 |
| electrical insp. | 2 | 5 | 4 | 2 |
| building insp. | 3 | 3 | 2 | 3 |

For some buildings all three inspections can be done. In that case the probability of finding a defect is $12 \%$, but the inspection takes as much time as the work of the longest-woking inspector needs. If only one inspector is sent, the probability of finding a defect decreases to $2.5 \%$. Write and solve the linear program allowing to find the optimal inspection plan for the upcoming week (i.e. how many buildings of what type should be inspected by which inspector and how many of them should be inspected by all inspectors). Your goal is to maximize the expected number of defects found by the inspectors.

LP11. (Based on [Ec79]) The banquet manager for the Aristocrat Hotel must select a soup, a salad, and an entree for a forthcoming banquet. The total cost of items selected must not exceed $\$ 4.25$ per serving. In addition,
the manager would like to maximize the total popularity rating of the items selected.

| Item | Cost per serving | Popularity rating | Remarks |
| :---: | :---: | :---: | :---: |
| Soups: | $\$ 0.35$ | 15 | Not compatible with salad C |
| A | 40 |  |  |
| B | $\$ 0.90$ |  |  |
| Salads: | $\$ 0.50$ | 25 |  |
| C | $\$ 0.75$ | 30 | Not compatible with entree F |
| D | $\$ 0.60$ | 50 |  |
| E | $\$ 1.20$ | 20 |  |
| F | $\$ 2.10$ | 70 |  |
| Entrees: | $\$ 3.40$ | 50 |  |
| G | $\$ 2.50$ |  |  |
| H |  |  |  |

Formulate an integer programming model for optimizing the banquet menu taking also remarks given in the table above as additional constraints. Solve it.

LP12. (Based on [LP05]) The Cleveland Sprinkler Company buys PVC pipes, which come in 120 cm lengths, and cuts them into the $30 \mathrm{~cm}, 42 \mathrm{~cm}$, and 56 cm lengths it requires for its projects. The following table gives the number of pieces of each required for each of the three lengths. Any cut of less than 30 cm is considered waste and is discarded. The same can be said about the additional pipes that exceed the demand. The company would like to purchase enough pipe to satisfy its requirements while minimizing its total losses.

$$
\begin{array}{c|c|c|c} 
& 30 \mathrm{~cm} & 42 \mathrm{~cm} & 56 \mathrm{~cm} \\
\hline \text { Required } & 1300 & 750 & 1530
\end{array}
$$

How many 120 cm pipes should be purchased to satisfy all the demands and how should they be cut. Formulate and solve an integer program allowing to find the solution.
Hint: The 120 cm pipes can be cut into several variations (e.g., four 30 cm lengths, two 30 cm lengths, and one 42 cm length with 18 cm of waste; one 30 cm length and two 42 cm lengths with 6 cm of waste; etc). The decision variables are the number of pipes cut into each of these configurations.

LP13. A sawmill produces standard boards which are 22 inches wide. Some clients order narrower boards (of the same length) though. The orders for today consist of 1207 -inch boards, 1355 -inch and 723 -inch ones. Smaller boards are cut off from the standard ones: e.g. the firm may decide to cut a standard board into 27 -inch boards, one 5 -inch and one 3 -inch one. It could also decide to cut it into 37 -inch boards, but in that case a 1 -inch strip is a waste. The firm wants to satisfy the demand in such a way that there is no waste produced. In this case it prefers to produce additional 7 -, 5 - or 3 -inch boards and store them so that they can be used on another day. The storage however induces storage cost proportional to the total sum of lengths of boards stored. Create the integer program allowing to find the way to cut the boards using least storage space and solve it.
Hint: Find all the possible ways to cut a 22 -inch board into 7 -, 5 - and 3 -inch ones without waste. The variables should be the numbers of 22 -inch boards cut using each way.

LP14. (Based on [Ec79]) National Transit operates buses between major cities and carries commercial packages on a space-available basis. A departing bus has room for up to 650 cubic feet of packages. Also, the packages that are included cannot exceed a total weight of 750 pounds. Finally, it has to be taken into account that half of the space is reserved for the priority packages and cannot be taken by non-priority ones. Packages awaiting
shipment are described in the table:

| Package | Priority | Volume $\left(\mathrm{ft}^{2}\right)$ | Weight (pounds) |
| :---: | :---: | :---: | :---: |
| 1 | Yes | 40 | 60 |
| 2 | Yes | 35 | 120 |
| 3 | No | 130 | 210 |
| 4 | Yes | 140 | 200 |
| 5 | No | 70 | 100 |
| 6 | No | 100 | 150 |
| 7 | No | 25 | 60 |
| 8 | No | 200 | 170 |
| 9 | Yes | 90 | 150 |
| 10 | No | 75 | 40 |

Formulate and solve an integer programming model for selecting packages to be included in the shipment. The goal of National Transit is to maximize the total number of packages shipped provided all the constraints are satisfied.

LP15. (Based on [Ec79]) Kronkhauser, Golden, and Smith, a public accounting firm, has received a request to audit the estate of a prominent individual who is being considered for appointment to a sensitive governmental position. The managing partner would like to assemble an auditing team that has maximum total auditing experience. At the same time, he does not want misunderstandings within the team, so he wants to have exactly two senior staff members on the team (as team's headperson and his deputy). The auditing team will travel by a private business jet that has a passenger-load capacity of 1000 pounds. Staff accountants who are available for assignment to the audit team are shown below:

| Name | Weight in pounds | Years of experience | Senior staff member |
| :---: | :---: | :---: | :---: |
| Linda Nelson | 95 | 2.0 | no |
| Susan Mayo | 120 | 6.5 | yes |
| Karen Dubronsky | 135 | 7.0 | yes |
| George Oswald | 140 | 5.0 | no |
| Andrew Goldman | 171 | 1.5 | no |
| William Masterton | 180 | 12 | yes |
| Roland Smithee | 185 | 5.5 | no |
| Donald Crowder | 190 | 2.5 | no |
| Ronald Kramer | 220 | 7.0 | no |
| Josh Zushinsky | 250 | 14 | yes |

Formulate and solve an integer program allowing to find the auditing team satisfying all the constraints.
LP16. A trading company must assign three sales managers to three sales offices. Its objective is to find the assignments that maximize the total yearly sales of all three offices. Naturally, only one person can be assigned to each sales office. The expected yearly sales (in millions of dollars) if each individual is assigned to each office are as follows:

|  | Atlanta | Boston | Chicago |
| :---: | :---: | :---: | :---: |
| Tardy | 21 | 25 | 17 |
| Vincent | 14 | 17 | 15 |
| Schuldiner | 21 | 19 | 13 |

The relocation expense budget for all three moves is $\$ 200000$. The costs (in thousands of dollars) of relocating each individual to each location are as follows:

|  | Atlanta | Boston | Chicago |
| :---: | :---: | :---: | :---: |
| Tardy | 65 | 50 | 30 |
| Vincent | 90 | 65 | 85 |
| Schuldiner | 100 | 20 | 80 |

Formulate and solve an integer programming model allowing you to decide which individual should be assigned to which office.

LP17. The head of Civil Defense in a town endangered by earthquakes wants to create the plan of assigning the casualties of a possible earthquake to four hospitals in town. The simulations suggests that almost all the
casualties will come from three neighbourhoods: the historical neighbourhood A (up to 250 people), the inner city B (up to 450 ), and the city center (C; up to 90 ). The times of transporting people to each hospital (in minutes) and their estimated capacities (in number of beds) are given below:

| Hospital | Beds available | Time from A | Time from B | Time from C |
| :---: | :---: | :---: | :---: | :---: |
| State Hospital | 400 | 25 | 23 | 20 |
| Saint Lucy | 120 | 12 | 16 | 8 |
| Saint Paul | 250 | 20 | 26 | 16 |
| Children's Hospital | 210 | 30 | 35 | 21 |

Write a linear program allowing you to find the assignment of casulaties to the hospitals, assuming that your goal is to minimize the expected number of minutes necessary to transport all the casualties to the hospitals.

LP18. A presidential candidate is down to the last five days of the campaign and the race is close. Only $\$ 300,000$ remains in the campaign budget. Four key states appear likely to swing the election one way or the other. In each state there are three campaign possibilities: it can be visited, a TV ad series can be purchased or no measures may be undertaken. The candidate's staff has made up the following estimates:

| State | Action | Percentage of votes gained | Days required | Cost |
| :---: | :---: | :---: | :---: | :---: |
| J | Visit | 8 | 4 | 200000 |
| J | Ad campaign | 3 | 0 | 100000 |
| K | Visit | 5 | 3 | 150000 |
| K | Ad campaign | 1 | 0 | 90000 |
| L | Visit | 3 | 1 | 40000 |
| L | Ad campaign | 4 | 0 | 15000 |
| O | Visit | 5 | 2 | 100000 |
| O | Ad campaign | 3 | 0 | 55000 |

In no case will both the ad series and a visit be scheduled in the same state. Formulate and solve an integer program allowing you to tell:
(a) Which states should be visited.
(b) In which states the ad series should be purchased.
(c) How much will the entire campaign cost.

The goal you should maximize is the average percentage of votes gained in the three states.
LP19. (Based on [NPTEL14]) Consider eight jobs (J1 to J8) and three non-identical parallel machines. The processing times are $23,8,15,7,5,19,16$ and 8 on M1, 20, $9,11,13,9,12,10$ and 9 on $\mathrm{M} 216,14,8,5,11,15,17$ and 13 on M3. Each job has to be allotted to only one machine. Formulate and solve an integer program allowing to find the allocation that minimizes makespan.

LP20. Consider the situation where there are eight projects but only three people bid for them. Person B does not bid for project 2. Find the assignment of people to projects that maximizes the total number of projects completed subject to the constraints that the total cost does not exceed $\$ 100000$ and that each person is assigned to at least two and at most four projects. The costs in $\$ 1000$ for each assignment are given in the table below:

|  | Person A | Person B | Person C |
| :--- | :---: | :---: | :---: |
| Project 1 | 12 | 15 | 8 |
| Project 2 | 16 | x | 7 |
| Project 3 | 14 | 15 | 21 |
| Project 4 | 13 | 14 | 9 |
| Project 5 | 17 | 13 | 11 |
| Project 6 | 22 | 24 | 16 |
| Project 7 | 13 | 12 | 10 |
| Project 8 | 9 | 6 | 12 |

Do it, using integer programming formulation of the problem.

LP21. (Based on [LP05]) Coolbike Industries manufactures boys' and girls' bicycles in both 20-inch and 26-inch models. Each week it must produce at least 350 girl models and 350 boys models. The following table gives the unit profit and the number of minutes required for production and assembly for each model:

| Bicycle | Unit profit | Production minutes | Assembly minutes |
| :---: | :---: | :---: | :---: |
| 20 - inch girls | $\$ 32$ | 28 | 20 |
| 20 - inch boys | $\$ 28$ | 25 | 16 |
| 26 - inch girls | $\$ 54$ | 19 | 30 |
| 26 - inch boys | $\$ 48$ | 26 | 25 |

The work is performed by two separate groups of workers: production team and assembly team, each of which works two (eight-hour) shifts per day, five days per week. The workers for both teams and both shifts are taken from the same pool of 20 people assigned to one of the teams for the whole week. Determine Coolbike's optimal (that is, maximizing profit) assignment of people into teams and optimal schedule for the week. What profit will it realize for the week? Solve the problem using a linear programming model.
LP22. (Based on [LP05]) Jones, Jimenez, and Sihota (JJS) is expanding its tax service business into the San Antonio area. The company wishes to be able to service at least 260 personal and 80 corporate accounts per week.
JJS plans to hire three levels of employees: CPAs, experienced accountants without a CPA, and junior accountants. The following table gives the weekly salary level as well as the projection of the expected number of accounts that can be serviced weekly by each level of employee.

| Employee | No. of pers. accounts | No. of corp. accounts | Salary |
| :---: | :---: | :---: | :---: |
| CPAs | 6 | 3 | $\$ 1500$ |
| Experienced accountants | 6 | 2 | $\$ 900$ |
| Junior accountants | 3 | 1 | $\$ 500$ |

JJS estimates that the number of candidates for work with a CPA will not exceed 10. Determine the number of employees from each experience level the firm should hire for its San Antonio office to minimize its total weekly payroll. Do it using an integer programming model.

LP23. (Based on [MK80]) The company providing cleaning services for the hospitals has found out that the demand for its services is quite seasonal. The number of hours of service needed in each month is given in the table below:

| Month | Hours |
| :--- | ---: |
| January | 1800 |
| February | 2500 |
| March | 1500 |
| April | 900 |
| May | 700 |
| June | 1000 |
| July | 700 |
| August | 600 |
| September | 1300 |
| October | 1600 |
| November | 1800 |
| December | 2300 |

Each person is hired at the begenning of a month for a 3 - or 6 -month contract. Each person needs to pass the training in the first month, which decreases the number of hours for which he/she is available to 80 . In any other month he/she can work for 196 hours. The monthly payment of any worker is $\$ 1700$. Write an integer program allowing you to find the optimal plan of hireing people for $3-$ and 6 -month contracts. Your goal should be the minimization of cost subject to covering all the demand.

LP24. (Based on [LP05]) The We-Haul Company is about to lease 5000 new trucks for its California operations. The
specifications of each truck under consideration are as follows:

| Truck | Country | Capacity | Capital outlay | Monthly lease |
| :---: | :---: | :---: | :---: | :---: |
| Ford | U.S.A. | 1 ton | $\$ 2000$ | $\$ 400$ |
| Chevrolet | U.S.A. | 1 ton | $\$ 1000$ | $\$ 700$ |
| Dodge | U.S.A. | 0.8 ton | $\$ 3500$ | $\$ 300$ |
| Mack | U.S.A. | 5 tons | $\$ 9000$ | $\$ 1000$ |
| Nissan | Japan | 0.5 ton | $\$ 1000$ | $\$ 200$ |
| Toyota | Japan | 0.9 ton | $\$ 0$ | $\$ 550$ |

We-Haul has decided that, for public relations reasons, given the current "Buy American" atmosphere, it will lease at least $60 \%$ of the trucks from American manufacturers. Each truck requires an initial capital outlay as well as monthly lease payments. We-Haul feels that it can support a total monthly lease payment of at most $\$ 3000000$ and total capital outlay of at most $\$ 1500000$. Its fleet requirements mandate at least a 8000 -ton total capacity for the 5000 trucks leased. Formulate and solve an integer program allowing you to determine the number of each truck We-Haul should lease to maximize its total capacity.
LP25. Consider a company making a single product. The demand for the next four days is 90, 200, 150 and 140 units. The company employs at most 15 people and each person works for 8 hours daily. Each person can make one unit of the product per hour. The employees can work for 2 extra hour per day overtime. The company uses different systems to pay for regular and overtime work. The regular work is paid $\$ 400$ per day of work, regardless of the amount of product made by the person (i.e. regardless of whether they have worked for the entire day or stayed idle for several hours. Cost of over time production is $\$ 70$ per product. The company has to meet the daily demand but can produce more and store them for further use at $\$ 10$ per unit per day. The company can get the product made outside and buy it at $\$ 125$ per unit. Find the number of peaople to be employed and the least cost production plan for those people.

LP26. (Based on [LP05]) Frank has just inherited $\$ 100000$. Frank is quite risk-averse, hence, the consulting company suggests the following potential investments that can offer good returns with small risk:

| Investment | Exp. return | Jones's rating | Liquidity anlysis | Risk factor |
| :---: | :---: | :---: | :---: | :---: |
| Savings account | $5.0 \%$ | A | Immediate | 0 |
| Certificate of deposit | $7.3 \%$ | A | 5-year | 5 |
| Atlantic Lighting | $9.8 \%$ | $\mathrm{~B}+$ | Immediate | 25 |
| Arcansas REIT | $12.8 \%$ | C | Immediate | 40 |
| Nocal Mining bond | $11.2 \%$ | $\mathrm{~B}+$ | 1-year | 15 |
| Minicomp Systems | $8.6 \%$ | B | Immediate | 50 |
| Antony Hotels | $9.5 \%$ | $\mathrm{~B}+$ | 1-year | 35 |

Given that Frank is risk-averse, the final recommendation should minimize the weighted average risk factor while meeting the additional goals:

- The weighted expected return should be at least $9 \%$.
- No more than $40 \%$ of the inheritance in investments rated below $\mathrm{B}+$ (that is, B or C ).
- At least $30 \%$ of the inheritance in immediately liquid investments.
- No more than $\$ 40000$ invested in savings account or certificate of deposit.

Find the portfolio recommended for Frank using a linear programming model.
LP27. The prison canteen has to provide food for the prison's 370 inmates. The meals it provides must consist of foods $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D in proportions which assure that they meet some state-imposed requirements related to their nutritional properties. The table below gives the amounts of nutritional elements that one unit of each food contains as well as the minimum requirements in each category that a meal for each person should satisfy:

|  | per unit of A | per unit of B | per unit of C | per unit of D | required per person |
| :--- | :---: | :---: | :---: | :---: | :---: |
| units of carbohydrates | 3 | 1 | 2 | 1 | 8 |
| units of vitamines | 4 | 2 | 6 | 3 | 23 |
| units of proteins | 1 | 2 | 5 | 7 | 12 |

Unit cost of food A is $\$ 20$, unit cost of food B is $\$ 30$ and they are available on the market in any amount. Product C is available at cost of $\$ 45$. Its availability is limited to 400 units. Finally, food D is available at $\$ 40$, but its availability is limited to 800 units

Find out using a linear programming model, how much of each type of food should be bought in order to minimize the cost while satisfying all the availability and nutritional constraints.

LP28. (Based on [LP05]) Tritech Mortgage specializes in making first, second, and even third trust deed loans on residential properties and first trust deeds on commercial properties. Any funds not invested in mortgages are invested in an interestbearing savings account. The following table gives the rate of return and the company's risk level for each possible type of loan:

| Loan type | Rate of return | Risk rating |
| :---: | :---: | :---: |
| First trust deeds | $7.25 \%$ | 3 |
| Second trust deeds | $12.25 \%$ | 6 |
| Third trust deeds | $15.75 \%$ | 9 |
| Commercial trust deeds | $13.5 \%$ | 5 |
| Savings account | $4.15 \%$ | 0 |

Tritech wishes to invest $\$ 38000000$ in available funding so that:

- Yearly return is maximized.
- At least $\$ 5000000$ is to be available in a savings account for emergencies.
- At least $50 \%$ of the money invested in trust deeds should be in first or second trust deeds.
- The average risk rating should not exceed 5 .

What distribution of funding do you recommend? What is the rate of return on this distribution of funds? Answer these questions using a properly constructed linear programming model.

LP29. (Based on [LP05]) Atlantic Standard Homes has aquired two lots in a new community in the Florida Keys: the first one is 10 acres, while the second is 15 acres. There are four models it can build on each lot, and Atlantic Standard must satisfy four requirements: at least 40 are to be one story; at least 50 are to have three or more bedrooms; all the houses built on the first lot are supposed to be one story; and there are to be at least 10 of each model. Atlantic Standard estimates the following gross profits:

| Model | Lot size (acres) | Stories | Bedrooms | Profit |
| :---: | :---: | :---: | :---: | :---: |
| Tropic | 0.20 | 1 | 2 | $\$ 40000$ |
| Sea Breaze | 0.27 | 1 | 3 | $\$ 46000$ |
| Orleans | 0.25 | 2 | 3 | $\$ 60000$ |
| Grand Key | 0.38 | 1 | 4 | $\$ 80000$ |

Formulate the problem as an integer linear programming model and solve for Atlantic Standards optimal production of homes in this community.

LP30. (Based on [NPTEL14]) Consider an airline that has demand for baggage handlers in a domestic airport. The hourly requirements for 24 hours starting from midnight are $7,5,3,1,2,6,7,9,10,13,16,19,15,10,10,13$, $18,20,23,22,15,9,8$ and 7 respectively. There are 6 shifts for handlers: the first shift starts at midninght. The next shifts start at 4AM, 8AM, 12, 4PM and 8PM. Each shift works for 8 consecutive hours. The demand is the same for all days. Using an integer programming model find the minimum number of handlers required that meets the demand. Suppose the starting time of the first shift can be moved to some other hour (with all other shifts starting 4, 8, 1216 and 20 hours after it). Will it make sense for the airline to change it, if the union accepts this change only if the payment of the baggage handlers increases by $10 \%$ in that case?

LP31. (Based on [NPTEL14]) Consider a caterer who has to provide food for several dinners happening in the next 8 days. The demand for cloth napkins that are used in the dinners is $89,130,109,100,70,150,206$ and 151. New napkin costs $\$ 8$. Napkins can be put to laundry and washed napkins can be used on subsequent days. Two types of laundry are available. The fast laundry that charges $\$ 2.9$ per unit and will deliver for use on the second day and the slow laundry that costs $\$ 1.3$ per unit and can deliver for use on the third day. Find the least cost purchase and use plan for the caterer using linear programming (the optimal solution should be integer without adding integrity constraint).

## References:

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