Theory and Methods of Optimization

Embedded Robotics

Computer assignments

NLP1. Par, Inc. is a small manufacturer of golf equipment and supplies whose management has decided to move into the market for medium- and high- priced golf bags. The steps involved in manufacturing a golf bag, management determined that each golf bag will require the following operations: cutting and dyeing the material, sewing,, finishing (inserting umbrella holder, club separators, etc.), inspection and packaging. The time required for each of these operations is given in the table below:

Department	Standard bag	Deluxe bag
Cutting and Dyeing	0.7	1
Sewing	0.5	0.85
Finishing	1	0.65
Inspection and Packaging	0.1	0.25

After studying departmental workload projections, the director of manufacturing estimates that 630 hours for cutting and dyeing, 600 hours for sewing, 708 hours for finishing, and 135 hours for inspection and packaging will be available for the production of golf bags during the next three months. Finally, the accounting department has estimated that after getting into account the demands for the two types of bags and the production costs, the profit (in USD) from selling S standard bags will be $80S - 0.1S^2$, while for D deluxe bags it will be $15D - 0.2D^2$. How many standard and how many deluxe bags should be produced by Par, Inc.? Find the answer by maximizing the total profit subject to the time constraints (plus nonnegativity constraints on D and S).

NLP2. (Based on [MOR17]) Consider three water consuming firms that belong to the same corporation. Each firm makes a product and water is needed in the process of making that product. Let the amounts of water consumed by each firm in kiloliters per hour be denoted by x_i, $i = 1, 2, 3$. The net benefits $B_i(x_i)$ from the production as functions of the amount of water used are given for each firm:

$$B_1(x_1) = -x_1^2 + 6x_1,$$

$$B_2(x_2) = -1.5x_2^2 + 7x_2,$$

$$B_3(x_3) = -0.5x_3^2 + 8x_3.$$

The available amount of water is 10kl/h. Moreover, the binding contracts for the firms require that at least 100 units of product 1 and at least 250 of product 2 must be produced, which requires x_1 to be at least 1kl/h and x_2 at least 3.75kl/h. Find the allocation of water to three firms that maximizes the total net benefit of the corporation from their production subject to the minimal production and water availability constraints.

NLP3. (Based on [BHM77]) An important problem in production management is the allocation of a given production quantity (determined by an aggregate model or by subjective managerial inputs) among a group of items. For example, let us assume that we have decided to produce $P = 8000$ units of a given product line consisting of three individual items. The allocation of the total quantity among the three items will be decided by the following mathematical model:

$$\begin{array}{ll} \text{minimize} & c = \sum_{i=1}^{3} \left(h_i \frac{Q_i}{2} + S_i \frac{d_i}{Q_i} \right) \\ \text{subject to} & \sum_{i=1}^{3} Q_i = P \end{array}$$

where:
Q_i is the production quantity for item i (in units),
h_i is inventory holding cost for item i (in \$ per month $\times$ unit),
S_i is the setup cost for item i (in \$),
d_i is the demand for item i (in units per month),
P is the total amount to be produced (in units).
Write a procedure allowing you to solve this problem. Apply it for the following values of the parameters: $h_1 = 1$, $h_2 = h_3 = 2$, $S_1 = 150$, $S_2 = 190$, $S_3 = 300$, $d_1 = 15000$, $d_2 = 45000$, $d_3 = 40000$.

NLP4. The throughput of a mobile terminal making use of three channels $k \in \{1, 2, 3\}$ for transmission can be computed using the formula:

$$T(p^1, p^2, p^3) = \sum_{k=1}^{3} \log_2 \left(1 + \frac{p^k h^k}{I_k}\right),$$

where $p^k \in \mathbb{R}^+$, $k = 1, 2, 3$, is the power used by the terminal for transmission on channel k (these are the variables of the model), while $h^k \in \mathbb{R}^+$, $k = 1, 2, 3$ are channel quality indicators and $I_k \in \mathbb{R}^+$, $k = 1, 2, 3$, are interferences experienced by the terminal on each channel (parameters of the model).

Write a program minimizing the total power consumption of the user subject to the minimum throughput constraint $T(p^1, p^2, p^3) \geq T_0 = 5GB/s$, where T_0 is the minimum throughput allowing for a satisfactory transmission. Assume the current values of h_ks and I_ks are known and given in the table below:

Channel	h_k	I_k
1	0.752	1.23
2	0.363	3.88
3	0.618	2.19

NLP5. The throughput of a mobile terminal making use of three channels $k \in \{1, 2, 3\}$ for transmission can be computed using the formula:

$$T(p^1, p^2, p^3) = \sum_{k=1}^{3} \log_2 \left(1 + \frac{p^k h^k}{I_k}\right),$$

where $p^k \in \mathbb{R}^+$, $k = 1, 2, 3$, is the power used by the terminal for transmission on channel k (these are the variables of the model), while $h^k \in \mathbb{R}^+$, $k = 1, 2, 3$ are channel quality indicators and $I_k \in \mathbb{R}^+$, $k = 1, 2, 3$, are interferences experienced by the terminal on each channel (parameters of the model).

Write a program maximizing the total throughput of the user subject to the total power constraint $p^1 + p^2 + p^3 = 1mW$, where T_0 is the minimum throughput allowing for a satisfactory transmission. Assume the current values of h_ks and I_ks are known and given in the table below:

Channel	h_k	I_k
1	0.752	1.23
2	0.363	3.88
3	0.618	2.19

NLP6. Solve the problem from the previous exercise with a modified objective function defined as his utility from transmission minus the total energy cost

$$u = T(p^1, p^2, p^3) - C \sum_{k=1}^{3} p^k$$

and the power constraint of the form $p^1 + p^2 + p^3 \leq 1mW$. Assume that the unit energy cost $C = 0.25$.

NLP7. (Based on [BHM77]) A balloon carrying an x-ray telescope and other scientific equipment must be designed and launched. A rough measure of performance can be expressed in terms of the height reached by the balloon and the weight of the equipment lifted. Clearly, the height itself is a function of the balloon's volume. From past experience, it has been concluded that a satisfactory performance function to be maximized is

$$P = f(V, W) = 100V - 0.4V^2 + 90W - 0.2W^2,$$

where V is the volume (in m^3, and W (in kg) the equipment weight. The project to be undertaken has a budget constraint of \$1040. The cost associated with the volume V is $2V$, and the cost of the equipment is $5W$. The minimum equipment weight is 20 kilos. Finally, in order to ensure that the balloon is able to fly to the stratosphere, where the equipment is needed, the equipment cannot be too heavy, that is – it needs to meet the constraint $2V \geq W + 30$. Find the optimal design in terms of volume and equipment weight.

NLP8. (Based on [BHM77]) A young R & D engineer at Carron Chemical Company has synthesized a sensational new fertilizer made of just three interchangeable basic raw materials. The company wants to take advantage of this opportunity and produce as much as possible of the new fertilizer. The company wants to take advantage of this opportunity and produce as much as possible of the new fertilizer. The company currently has \$40000 to buy raw materials at a unit price of \$800, \$400 and \$500 per unit, respectively. The supply of the first and the second raw material are limited by 30 and 50 units respectively. When amounts x_1 x_2 and x_3 of the basic raw materials are combined, a quantity q of fertilizer results given by:

$$q = 3x_1 + 2x_2 + 2x_3^2 - 0.4x_1^2 - 0.2x_2^2 - 0.45x_3^2 - 0.1x_1x_3,$$

which is then sold for \$1400 per unit. Formulate and solve the quadratic programming problem allowing to find the amounts of each raw material that should be bought.

NLP9. A chain of stores wants to build a distribution center to provide its retail outlets with articles to sell. The data available are the locations of the retail outlets (x_i, y_i), $i = 1, \dots, 8$ and the amounts of articles sold at each outlet w_i, $i = 1, \dots, 8$. Find the location for the distribution center (x_0, y_0) minimizing the weighted sum of Euclidean distances from the center to the outlets (with w_i serving as weights) satisfying an additional constraint that the location is no more than 10 kilometers away from the freeway whose position is given by the formula $y = 50+1.3x$, $x \in [-20, 40]$, which translates to the following constraint: $y \in [44+1.3x, 56+1.3x]$. The data for the problem is given in the table below.

Outlet number	x_i	y_i	w_i
1	−19	65	4200
2	37	180	2100
3	149	10	1500
4	87	235	6500
5	−52	−12	4000
6	10	−64	1200
7	25	−11	1500
8	45	−121	3000

NLP10. A chain of stores wants to build a distribution center to provide its retail outlets with articles to sell. The data available are the locations of the retail outlets (x_i, y_i), $i = 1, \dots, 8$ and the amounts of articles sold at each outlet w_i, $i = 1, \dots, 8$ as well as the locations of production centers (x_i^P, y_i^P), $i = 1, 2$. Find the location for the distribution center (x_0, y_0), which minimizes the weighted sum of Euclidean distances from the center to the outlets (with w_i serving as weights) and keeps the sum of distances from the production centers below 100. The data for the problem is given in the tables below.

Outlet number	x_i	y_i	w_i
1	−19	65	4200
2	37	180	2100
3	149	10	1500
4	87	235	6500
5	−52	−12	4000
6	10	−64	1200
7	25	−11	1500
8	45	−121	3000

Prod. center number	x_i^P	y_i^P
1	36	60
2	−28	20

NLP11. A chain of stores wants to build a distribution center to provide its retail outlets with articles to sell. The data available are the locations of the retail outlets (x_i, y_i), $i = 1, \dots, 8$ and the amounts of articles sold at each outlet w_i, $i = 1, \dots, 8$ as well as the locations of production centers (x_i^P, y_i^P), $i = 1, 2$. Find the location for the distribution center (x_0, y_0), which minimizes the weighted sum of Euclidean distances from the center to the outlets (with w_i serving as weights) and keeps the distances from each of the production centers below

100. The data for the problem is given in the tables below.

Outlet number	x_i	y_i	w_i
1	-19	65	4200
2	37	180	2100
3	149	10	1500
4	87	235	6500
5	-52	-12	4000
6	10	-64	1200
7	25	-11	1500
8	45	-121	3000

Prod. center number	x_i^P	y_i^P
1	36	60
2	-28	20

NLP12. Solve a modification of the distribution center location problem from the previous exercise where, instead of choosing a location minimizing the weighted sum of distances between the distribution center and the outlets subject to the constraints about the distances from the production centers, we do the unconstrained optimization of weighted sum of distances between the distribution center and the outlets as well as the distances between the distribution and the production centers. The weights for the production centers should be $w_1^P = 8000$ and $w_2^P = 4000$.

NLP13. Suppose a telecommunication company tries to find the location for a transmission tower. The tower is to provide for the area including several towns in the Cincinnati area. The data given below enumerates them together with their locations on the grid (the S-N and W-E distances are given in miles)

Town	x_i	y_i
Cincinnati	17	14
Florence	10	10
Covington	12	16
Evendale	12	22
Fairfax	13	17
Milford	19	19

The quality of the transmission depends crucially on the distance between the tower and any mobile. Therefore, the goal of the company is to put the tower in a spot that minimizes the maximum Euclidean distance between the tower and any of the towns that are supposed to be covered. This is a minmax problem so it can be transformed into a constrained minimization problem just like in exercise 6 on the problem set 1. Formulate and solve this problem.

NLP14. (Based on [Wi80]) The daily cost of energy-production in an electric plant consisting of 2 units is given by:

$$\sum_{i=1}^{2} c_i x_i + \sum_{i=1}^{2} d_i x_i^2,$$

where $c_1 = 7.5$, $c_2 = 4.4$, $d_1 = 13.6$ and $d_2 = 1.9$ are known nonnegative factors, while x_1 and x_2 are outputs of units in megawatts.

The system output $x_1 + x_2$ must cover the energy demand of $D = 18000$ megawatts plus the transmission losses

$$P_L = \sum_{i=1}^{2} \sum_{j=1}^{2} H_{ij} x_i x_j.$$

where all the values H_{ij} are given by the matrix $H = \begin{bmatrix} 6 & 8 \\ 8 & 24 \end{bmatrix} \times 10^{-6}$.

Finally, each unit has a known operating range:

$$5000 \leq x_1 \leq 16000, \quad 8000 \leq x_2 \leq 20000$$

Find the minimum of the total operating cost subject to all the constraints enumerated above.

NLP15. Solve the modification of the problem from the previous exercise, where we maximize the system output minus the transmission losses $x_1 + x_2 - P_L$ subject to the constraint on the energy-production cost of the form

$$\sum_{i=1}^{2} c_i x_i + \sum_{i=1}^{2} d_i x_i^2 \leq 7 \times 10^7$$

plus the operating range constraints given in the previous exercise.

NLP16. (Based on [BM68]) The variables to be determined are x_{ij} ($i = 1, 2, 3$ and $j = 1, 2$), the numbers of weapons of types i assigned to targets j. The probabilities a_{ij} that target j will be undamaged by an attack using one unit of weapon i are given in the table below:

	w. 1	w. 2	w. 3
t. 1	0.35	0.97	0.76
t. 2	0.55	0.89	0.85

Constraints on the amount of damage to be inflicted on the targets are of the form

$$1 - \Pi_{i=1}^{3} a_{ij}^{x_{ij}} \geq d_j, \quad j = 1, 2$$

with $d_1 = 0.9$ and $d_2 = 0.8$. For purposes of formulating the objective function, let us assume that the cost of weapons assignment is quadratic over the range being considered and equal to $c_i(x_{i1} + x_{i2}) + \widetilde{c}_i\,(x_{i1} + x_{i2})^2$, with values of c_i, $\widetilde{c}_i$, $i = 1, \ldots, 3$ given in the table:

	w. 1	w. 2	w. 3
c_i	10	12	15
$\widetilde{c}_i$	1.92	0.35	0.85

We need to choose x_{ij}s to minimize the total cost of all the weapons used subject to nonnegativity constraints and constraints on the amount of damage on the targets. Although this is not realistic, assume that the amounts of weapons need not be integers. Notice that the constraints can be reformulated in such a way that they all become linear.

NLP17. For the data from the previous exercise choose the assignment of weapons to targets maximizing the sum of probabilities that each target will be damaged

$$\sum_{j=1}^{2} \left[1 - \Pi_{i=1}^{3} a_{ij}^{x_{ij}}\right]$$

subject to the following constraints on the number of weapons used for each target

$$\sum_{j=1}^{2} x_{ij} \leq 20, \quad i = 1, 2, 3, \qquad \sum_{i=1}^{3} x_{ij} \leq 40, \quad j = 1, 2$$

plus the nonnegativity constraints.

NLP18. An aircraft producer is seeking for a design of the airframe with the lowest production cost satisfying several functional constraints. The cost is given by

$$c(x_1, x_2, x_3) = 250 x_1^{0.3} x_2^{0.25} x_3^{0.1}$$

with x_1 denoting airframe weight, x_2 – propeller weight and x_3 – the length of the airframe. The functional constraints are the following: The length of the airframe must be between 90 and 150 meters. The relations between the airframe's length and weight and between the weights of the airframe and that of the propeller which have to be satisfied are the following: $50x_3 \leq x_1 \leq 75x_3$ and $2x_1 \leq x_2 \leq 3x_1$. Finally, a mass fraction constraint of the form

$$0.4 \leq \frac{0.5x_1 + 20000}{0.5x_1 + x_2 + 20000} \leq 0.55$$

is required to obtain good performance of the aircraft.

Write a program minimizing the airframe production cost subject to all the constraints.

NLP19. An investor tries to figure out which whether it makes sense to invest in the tobacco industry. He knows that the number of employees available in the local market is very small, so he thinks it only makes sense to invest in an industry with a positive capital output elasticity/labor output elasticity ratio. He thus needs to estimate the elasticities for the industry from the avalibale data. It is known that the following Cobb-Douglas formula should approximately hold in this case:

$$y = \alpha c^{\beta_1} l^{\beta_2},$$

where l is the labor input (the total number of person-hours worked in a year), c is the capital input (the real value of all machinery, equipment, and buildings), y is the total production (these values are known for many firms in each industry), while α (productivity factor), β_1 (capital output elasticity) and β_2 (labor output elasticity) are unknown.

Estimate α, β_1 and β_2 from the following experimental data by minimizing the sum of the squares of the deviations between the experimental and predicted values of y:

y	c	l
75	7.0	10.5
36.8	8.3	6.2
142.5	2.5	18.0
171	15.0	13.5
131	21.5	9.2
1.7	4.8	0.5

NLP20. (Based on [JK05]) Michaelis-Menten enzyme kinetics is described by the following equation:

$$v = \frac{V_{\max} S}{K_M + S},$$

where S is the concentration of substrate, v is the initial velocity of reaction, $V_{\max}$ is the saturation velocity and K_M is Michaelis-Menten constant. Write a procedure to determine $V_{\max}$ and K_M from a vector of measurements of v as function of S which will minimize the least-square error

$$Err = \sum_{i=1}^{N} \left(v_i - \frac{V_{\max} S_i}{K_M + S_i} \right)^2.$$

Apply it to determine the values of $V_{\max}$ and K_M for the data given in the table:

S	0.25	0.3	0.4	0.5	0.7	1.0	1.4	2.0
v	2.4	2.6	4.2	3.8	6.2	7.4	10.2	11.4

NLP21. (Based on [BM68]) The standard way to solve the linear regression problems is by minimizing the mean quadratic error (which can be done efficiently in several ways). In some problems it is advised to use some other error measures. The one of interest here is the sum of pth powers of deviations

$$\sum_{i=1}^{m} |\alpha_i|^p \tag{1}$$

computed as

$$\alpha_i = x_{i1} b_1 + \ldots + x_{in} b_n - y_i, \quad \text{for } i = 1, \ldots, m. \tag{2}$$

The problem here is for given sets of measurements $x_{i1}, \ldots, x_{in}$, $i = 1, \ldots, m$ to find $b_1, \ldots, b_n$ minimizing (1) subject to (2).

For even p this can be written equivalently as follows:

$$\text{minimize} \sum_{i=1}^{m} \alpha_i^p$$

subject to

$$-y_i + x_{i1} b_1 + \ldots + x_{in} b_n + \alpha_i = 0, \quad i = 1, \ldots, m.$$

Solve (the last formulation of) the problem for $p = 3$ for the following data:

i	y_i	x_{i1}	x_{i2}	x_{i3}
1	99	1	8.5	106
2	89	1	8.2	72
3	103	2	9.5	75
4	86	1	7.4	99
5	91	1	7.0	165
6	95	1	8.0	180
7	100	1	9.5	99
8	135	2	12.3	210
9	112	2	10.8	175
10	81	1	7.0	120
11	89	1	8.2	132
12	97	1	9.0	157
13	113	1	15.7	75
14	51	0.5	3.7	80
15	75	1	6.2	105
16	98	1	10.6	205
17	100	2	11.4	109
18	94	2	7.1	90
19	121	2	13.0	87
20	39	0.5	3.4	150

NLP22. (Based on [BM68]) The standard way to solve the linear regression problems is by minimizing the mean quadratic error (which can be done efficiently in several ways). In some problems it is advised to use some other error measures. The one of interest here is the sum of pth powers of deviations

$$\sum_{i=1}^{m} |\alpha_i|^p \tag{3}$$

computed as

$$\alpha_i = x_{i1}b_1 + \ldots + x_{in}b_n - y_i, \quad \text{for } i = 1, \ldots, m. \tag{4}$$

The problem here is for given sets of measurements $x_{i1}, \ldots, x_{in}$, $i = 1, \ldots, m$ to find $b_1, \ldots, b_n$ minimizing (3) subject to (4).

For odd p this can be written equivalently as follows:

$$\text{minimize} \sum_{i=1}^{m} \alpha_i^p$$

subject to

$$\begin{aligned} -y_i + x_{i1}b_1 + \ldots + x_{in}b_n + \alpha_i \geq 0, &\quad i = 1, \ldots, m, \\ y_i - (x_{i1}b_1 + \ldots + x_{in}b_n) + \alpha_i \geq 0, &\quad i = 1, \ldots, m, \end{aligned}$$

Solve (the last formulation of) the problem for $p = 3$ for the data given in the previous excercise.

NLP23. The standard way to solve the linear regression problems is by minimizing the mean quadratic error. In some problems it is advised to use some other error measures. One of the most popular ones is the maximum absolute deviation

$$\alpha = \max_{i=1,\ldots,m} |x_{i1}b_1 + \ldots + x_{in}b_n - y_i| \,. \tag{5}$$

The problem is thus for a given sets of measurements $x_{i1}, \ldots, x_{in}$, $i = 1, \ldots, m$ to find $b_1, \ldots, b_n$ minimizing α.

This can be done as follows:

$$\text{minimize } \alpha$$

subject to

$$\begin{aligned} -y_i + x_{i1}b_1 + \ldots + x_{in}b_n + \alpha \geq 0, &\quad i = 1, \ldots, m, \\ y_i - (x_{i1}b_1 + \ldots + x_{in}b_n) + \alpha \geq 0, &\quad i = 1, \ldots, m. \end{aligned}$$

Solve this problem for the data given in excercise NLP21.

NLP24. (Based on [BM68]) The standard way to solve the linear regression problems is by minimizing the mean quadratic error (which can be done efficiently in several ways). In some problems the relation between the variables is not linear, hence the methods used for linear problems have a limited application. Suppose we expect that the relation between variables x_{ij}, and the variables y_i, $i = 1, \ldots, m$ is the following:

$$y_i \approx b_0 + b_1 e^{x_{i1}} + \ldots + b_n e^{x_{in}}, \quad i = 1, \ldots, m$$

and we want to find the coefficients b_0, b_1,. . . ,b_n. To do that for given sets of measurements $x_{i1}, \ldots, x_{in}$, $i = 1, \ldots, m$ we need to find $b_0, \ldots, b_n$ and $\alpha_1, \ldots, \alpha_m$ minimizing

$$\text{minimize} \sum_{i=1}^{m} \alpha_i^2$$

subject to

$$-y_i + b_0 + b_1 e^{x_{i1}} + \ldots + b_n e^{x_{in}} + \alpha_i = 0, \quad i = 1, \ldots, m.$$

Solve this problem for the data given in excercise NLP21.

NLP25. Solve the problem combining the problems given in the last two exercises, that is, find the exponential model as in the last exercise, but minimizing the maximum absolute deviation. This can be done by solving the following problem:

$$\text{minimize } \alpha$$

subject to

$$-y_i + b_0 + b_1 e^{x_{i1}} + \ldots + b_n e^{x_{in}} + \alpha \geq 0, \quad i = 1, \ldots, m,$$
$$y_i - b_0 - b_1 e^{x_{i1}} - \ldots - b_n e^{x_{in}} + \alpha \geq 0, \quad i = 1, \ldots, m,$$

Solve this problem for the data given in excercise NLP21.

NLP26. An environmental scientist wants to find out whether the temperatures in his country have an increasing trend. In order to do that he tries to fit the curve (showing theoretical dependence of average monthly temperature T on time in months t)

$$T(t) = \alpha \sin\left(\frac{2\pi(t - t_0)}{12}\right) + \beta t + \gamma$$

(with α, β ,γ and t_0 being the parameters of the curve) to the average monthly temperatures given in the table below:

month	jan	feb	mar	apr	may	jun	jul	aug	sep	oct	nov	dec
2013	1.0	1.3	3.5	7.8	12.5	15.1	17.9	15.7	12.9	9.1	4.5	2.1
2014	1.3	0.7	3.2	6.7	12.7	14.9	15.9	16.3	14.5	9.0	4.1	1.2
2015	0.1	1.2	3.8	5.3	11.5	14.5	16.1	17.8	15.1	8.0	3.2	1.9
2016	0.9	1.6	5.1	7.9	10.2	14.2	16.3	19.8	12.5	9.3	6.0	3.2
2017	1.9	2.3	4.5	8.4	12.0	15.9	17.8	18.9	13.0	8.7	6.3	3.0
2018	2.1	0.2	3.9	7.3	11.6	15.7	18.5	19.9	12.7	8.1	4.2	2.8
2019	1.9	0.9	4.5	8.9	13.8	16.8	17.9	18.1	15.6	7.4	6.9	2.3
2020	1.5	1.5	5.4	9.1	15.1	15.3	18.8	20.0	14.1	8.5	5.3	1.7
2021	2.4	1.7	5.0	6.1	13.7	17.2	20.1	18.7	14.8	7.9	7.7	3.5
2022	1.4	2.6	5.9	8.4	12.9	18.3	19.7	20.5	15.7	9.6	3.7	2.6

Write a procedure allowing you to find the parameters of the function T minimizing the sum of the squares of the deviations between the real and the predicted (given by T) values of the temperatures.

NLP27. Solve the variant of the problem from the previous exercise, this time assuming that it is not the average temperature but the amplitude that changes linearly with time. Hence, try to fit the curve

$$T(t) = (\alpha + \beta(t - t_0)) \sin\left(\frac{2\pi(t - t_0)}{12}\right) + \gamma$$

(with α, β ,γ and t_0 being the parameters of the curve) to the average monthly temperatures given in the table in the previous exercise. Do it by writing a procedure allowing you to find the parameters of the function T minimizing the sum of the squares of the deviations between the real and the predicted (given by T) values of the temperatures.

NLP28. Write a procedure to find a distance between any two polygones in $\mathbb{R}^2$ defined by the sets of their vertices (given vy the user). The way to write the problem as a quadratic programming problem can be found in [Sch02]. Test it for some arbitrary polygones.

NLP29. Write a procedure to find the smallest ball (circle) in $\mathbb{R}^2$ enclosing a given set of points. The way to write the problem as a quadratic programming problem can be found in [Sch02]. Test it for some arbitarry sets of points.

NLP30. The cost of refined oil when shipped via the Malacca Straits to Japan in dollars per kiloliter was given in [U68] as the linear sum of the crude oil cost, the insurance, customs, freight cost for the oil, loading and unloading cost, sea berth cost, submarine pipe cost, storage cost, tank area cost, refining cost, and freight cost of products as

$$\begin{aligned} c &= c_c + c_i + c_x + \frac{2.09 \times 10^4 t^{-0.3017} + 5.042 \times 10^3 q^{-0.1899} + 0.1049 q^{0.671}}{360} \\ &+ \frac{1.064 \times 10^6 a t^{0.4925} + 4.242 \times 10^4 a t^{0.7952} + 1.813 i p (nt + 1.2q)^{0.861} + 4.25 \times 10^3 a (nt + 1.2q)}{52.47 q (360)} \end{aligned}$$

where a is annual fixed charges percentage, c_c is crude oil price (in $\$/kl$), c_i – insurance cost (in $\$/kl$), c_x – customs cost (in $\$/kl$), i – interest rate, n – number of ports, p – land price (in $\$/m^2$), q – refinery capacity (in bbl/day) and t – tanker size (in kl). Write a procedure to compute the minimum cost of oil and the optimum tanker size t and refinery size q by Newton's method using Armijo rule to choose the stepsize. Apply it to the following data:

a	c_c	c_i	c_x	i	n	p
0.20	12.5	0.5	0.9	0.1	2	7000

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