Method to characterize collective impact of factors on indoor air

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HIGHLIGHTS

- We characterized the joint impact of a collection of factors on indoor air quality.
- We analyzed real data: temperature, relative humidity and CO\textsubscript{2} concentration.
- The mean square displacement is used as a measure to characterize indoor air.
- We identified three types of the joint impact: retarding, stabilizing and promoting.

ABSTRACT

One of the most important problems in studies of building environment is a description of how it is influenced by various dynamically changing factors. In this paper we characterized the joint impact of a collection of factors on indoor air quality (IAQ). We assumed that the influence is reflected in the temporal variability of IAQ parameters and may be deduced from it. The proposed method utilizes mean square displacement (MSD) analysis which was originally developed for studying the dynamics in various systems. Based on the MSD time-dependence descriptor $\beta$, we distinguished three types of the collective impact of factors on IAQ: retarding, stabilizing and promoting. We presented how the aggregated factors influence the temperature, relative humidity and CO\textsubscript{2} concentration, as these parameters are informative for the condition of indoor air. We discovered, that during a model day there are encountered one, two or even three types of influence. The presented method allows us to study the impacts from the perspective of the dynamics of indoor air.

1. Introduction

In science and technique, there are very often considered objects whose properties are strongly affected by external factors. Many reasons cause a need to characterize their influence, and different strategies can be used to accomplish this task. The most straightforward approach is based on separation of individual factors and determination of their influence on the properties of tested object. This strategy is attractive cognitively. However, in cases when factors are numerous,
different in character and dynamically interacting with each other, the impact of individual ones is very difficult to extract and describe. Even if achieved, the description has a low practical value, because an isolated influence of a single factor is rarely encountered in real conditions. Therefore, it is well justified as well as more promising to perform the analysis of the collective impact of many factors on parameters describing the object. This approach better corresponds to reality and may additionally provide the information on the types of factors which have dominating influence on the objects behavior. This position was taken in our study. The aim of this work is to propose a method to characterize the collective influence of numerous factors on quantities describing the object. This approach better corresponds to reality and may additionally provide the information on the types of factors which have dominating influence on the objects behavior.

Indoor air is one of the most important elements of human environment [1]. In industrialized countries people spent about 90% of their lifetime indoors. Numerous studies have shown that indoor air quality (IAQ) has a strong impact on occupant health, productivity and perception of comfort [2,3]. Recently IAQ became a serious problem in many countries and there are sound reasons to believe the importance of this problem will be intensified in the near future [4]. Therefore, considerable attention has been directed towards methods of IAQ evaluation. Different aspects can be taken into account in this task. It is however one of major general concerns to develop a method of examining the influence of various factors on the properties of air inside an enclosed space.

Usually, IAQ is characterized by a set of time-dependent physical and chemical parameters e.g. temperature (T), relative humidity (RH), concentration of air contaminants [5,6]. In time, these quantities are under complex influence of many factors. They are associated with outdoor conditions (e.g. meteorology) [7,8], interactions between a building and its surrounding (e.g. via infiltration) [9,10], individual properties of the building (materials, structure, construction) [11,12], heating, ventilation and air conditioning systems (HVAC) [13,14], indoor space arrangement (space organization, furnishing, equipment and appliances) [15], sources of heat and pollutants emission [16,17] as well as patterns of occupants activity [18,19]. These numerous factors, in various combinations, simultaneously affect IAQ parameters. This process is very complex. In principle, the factors do not act independently, but they enter nearly unlimited interactions, which dynamically change in time. For that reason, it is difficult to quantify the influence and relative importance of the individual factors.

The proposed approach to characterize a collective influence of factors on indoor air is based on Mean Square Displacement (MSD) analysis. The method was applied to the time series of indoor air parameters. We focused our attention on three, fundamental types of impacts: promoting (enhancing), stabilizing and retarding the temporal variation of these quantities. Our study was motivated by a fact, that the proposed approach can be applied to study various complex systems which are affected by numerous phenomena, that are difficult for identification and quantification individually. The aspect of the applicability of the proposed method was also important in this work.

2. Methods

In order to reach the goal of this study several assumptions were made. They were reasonable in view of the experimental results and mathematical methods applied.

2.1. Mean square displacement analysis

A tool, which we proposed for recognizing the influences which promote, retard or stabilize the variation of indoor air parameters is mean square displacement (MSD) analysis. The term mean square displacement originates from modeling the diffusion process. The MSD, in a traditional meaning, is a measure of the average square distance that a molecule travels. It is defined by the following formula [20,21]:

\[
M(t) = \langle (X_t - X_0)^2 \rangle .
\]  

(1)

In this equation, \( \{X_t\} \) is a process describing the molecule position; therefore \( X_t - X_0 \) is the displacement of the molecule between time 0 and \( t \). The final MSD at time \( t \) is obtained as averaged (as indicated by the angle brackets) squared-displacement over all molecules’ positions at time \( t \) in the system.

In some applications, instead of MSD defined as an average square displacement (1), a sample MSD is considered. The sample MSD for the sample \( \{X_i, \ i = 1, 2, \ldots, N\} \) being a realization of the process with stationary increments is defined as follows [22]:

\[
M_N(\tau) = \frac{1}{N - \tau} \sum_{k=1}^{N-\tau} (X_{k+\tau} - X_k)^2 .
\]  

(2)

The sample MSD is a time average MSD for a sample of length \( N \), regarded as a function of \( \tau \), which points at the difference between observations. The sample MSD defined as in Eq. (2) is a random variable. The diffusion regime can be expressed in terms of MSD defined in (1) [23]. More precisely, we call the process \( \{X_t\} \) as anomalous diffusive if it has non-linear MSD:

\[
M(t) \sim t^\beta .
\]  

(3)

Moreover, when \( \beta > 1 \), the process exhibits superdiffusive behavior, while for \( \beta < 1 \)–it is subdiffusive. For \( \beta = 1 \) the process reduces to normal diffusion. Some examples of anomalous diffusion processes one can find in Refs. [24–27].
anomalous diffusion property can be also expressed by means of sample MSD defined in (2). Namely, very often anomalous diffusive process is defined as a system for which the sample MSD exhibits a power law behavior [28–30]:

\[ M_N(\tau) \sim \tau^\beta. \] (4)

And, similar as previous, for \( \beta > 1 \) we have superdiffusive behavior, for \( \beta < 1 \) — subdiffusive and for \( \beta = 1 \) we have normal diffusion. In this case the \( \beta \) parameter specifies the type of diffusion. We should mention here there are many processes that exhibit super and sub-diffusive behavior. For superdiffusion we should emphasize for example Lévy walks [31,32] while for subdiffusion — fractional Brownian motion [33,34], continuous time random walk [35], heterogeneous diffusion process [36,37], scaled Brownian motion [38]. For other processes with subdiffusive property we refer the reader to Refs. [39–42].

The three ranges of \( \beta \) values distinguished in statistical physics: \( \beta = 1, \beta < 1 \) and \( \beta > 1 \) [43,44] may be associated with different kinds of predominant influences in the systems. The linear growth of MSD with time (\( \beta = 1 \)) is characteristic for the homogeneous systems with stationary conditions and random fluctuations. It is typical for the processes, which are local in space and do not have any memory of the system’s history. One may think of it in terms of a system where the variation is stabilized. However, in many complex, inhomogeneous systems sample mean square displacement grows sub-linearly (\( \beta < 1 \)) or super-linearly (\( \beta > 1 \)) with time. Basically, the sub-linear dynamics (\( \beta < 1 \)) is observed in the disordered systems, which are in stable conditions, possess memory (include history dependence), exhibit long-range correlation. In these systems, changes are retarded due to the presence of either mobile or immobile obstacles or by complex physical/chemical interactions. On the other hand, the super-linear growth \( \beta > 1 \) is observed when the system is unstable, in a transient state. It is frequently associated with the promoting impacts e.g. the directed motion, as in the case of active transport processes.

2.2. A method of collective impact characterization

The method proposed in this work involves several assumptions.

Indoor air is affected by many factors, which are difficult to separate. Therefore, their influence should be characterized in terms of a collective impact. This is an objective of the presented work. Our strategy was based on the measurement data. It is the most reliable source of information on indoor air. In order to ensure the completeness of information, measurements were conducted over relatively long time, e.g. several months. The key issue was the appropriate resolution of the recorded data. The need to assure the applicability of the proposed approach caused that the measurements were performed using traditional, well-known methods and instruments.

The combined influence of factors on IAQ was characterized in time domain. The justification of this approach was based on the following reasoning. Factors, which have an impact on indoor air, are subject to time change. Hence, their influence also alters with time. The temporal variation of factors results in the corresponding changes of indoor air parameters. Due to fact that indoor air is affected by time-dependent factors, the information about collective impact may be extracted from univariate time series of indoor air parameters. We chose temperature, relative humidity and CO\(_2\) concentration as the parameters which are descriptive for IAQ.

The information about the collective impact is included in characteristic variations of the mentioned parameters over time. These changes are contained in the appropriate segments of the time series, which may be delimited by time. Hence, the combined influence of factors on IAQ was characterized for the deliberately selected time periods. There shall be good reasons to believe that in a single time period the character of influence is the same. In our work, the time periods were chosen on the basis of two criteria: type of day and hour within day. Two kinds of days were distinguished: working days and days off. The analysis was performed separately for working days and days off, in order to compare the categories of influence upon the existence of factors associated with human presence and its lack.

Usually, indoor air is affected by factors in a periodic way. This fact is reflected in time series of indoor air parameters. Namely, they also display cyclic changes. The basic period of their variation is 24 h. For this reason, the evolution of collective influence was examined over the period of 24 h.

Every day was divided into twenty-four hours, starting from 00:00 a.m. We assumed that during one hour, the impact of factors is stable enough, to evoke a consistent character of temporal variation of indoor air parameters. It could be characterized by the type of the associated power–law dependence. In view of the daily cycle of factors change, we proposed a thesis that the character of temporal variation is similar on the same hour of different days. For this reason, the measurement data collected on different days, but at the same time of day, were analyzed jointly, combined into a one time series. We assumed that the observations made on the same hour of days of the same kind do not comprise events with significantly different characteristics. The collective influence of various factors on IAQ parameters variation was determined for sets of such data segments. More precisely, the power-law time-dependence was determined individually for each set. The period of one hour was chosen arbitrarily, but we considered the necessity of assuring a sufficiently large data set for MSD analysis as well as the possibility of interpreting the obtained results.

We assumed that the collective influence of various factors is reflected in the variability of indoor air parameters. Based on earlier works [45,46] the variation of indoor air parameters may be related to the impact of various factors just like diffusion in physical systems occurs through molecular collisions. For this reason, the MSD time dependence was explored for characterizing the collective influence of factors on indoor air.
In this work there was considered sample MSD; see (2) and (4). MSD analysis was performed for each indoor air parameter individually. Since the data was recorded only once over some relatively long time period the analysis was based only on one trajectory for each particular indoor air parameter. The sample MSD was determined for a time series, which was obtained from the univariate time series of the measurement data, after a sequence of transformations; see Fig. 1. First, the original time series was differentiated. Following, it was divided into subseries, which were one-day long. Further, each one-day long subseries was divided into fragments, which were one-hour long. In the next step, there were formed new time series. They resulted from combining one-hour long subseries associated with the same hour on distinct days. There were obtained 24 such time series, each assigned to different hour of a 24 h period (from 00:00 to 23:00), aggregated over all days. These time series were individually analyzed using MSD, leading to \( \beta \) calculation for the data associated with each hour of the day.

It is requested, before applying sample MSD analysis, to check the stationarity of increments of the examined system. There are methods available that can be useful to address this problem [47,48]. In order to show that the sample \( \{X_i, i = 1, 2, \ldots, N\} \) comes from the process with stationary increments, in this paper, we proposed to analyze the sample autocorrelation (ACF) and partial autocorrelation (PACF) functions of increments of the sample [49]. If those measures exhibit behavior adequate to stationary processes (i.e. tend to zero with respect to the lag or have zero values after some specific lag), then we could expect the increments of the examined process are stationary.

It is required, before the examination of sample MSD in terms of \( \beta \) over one-hour periods over a day, that the relationship given by (4) is acknowledged. In general, fitting may be attempted if the data set is sufficiently large (greater than 1000 elements). In our analysis we preceded the actual calculations of \( \beta \) by checking the shape of sample MSD time-dependence for the original data set and its subsets. The aim of this step was to find the range of the data set size where the linear fit is possible and the obtained \( \beta \) estimate is constant. Based on that, the consistency of the characterization of factors influence on IAQ for a particular time period, using \( \beta \) may be confirmed.

There are distinguished three types of the collective influence of factors on particular indoor air parameters. They are indicated by different ranges of \( \beta \). The first type of influence is described as retarding the variation of indoor air parameters. The corresponding range of \( \beta \) is \( \beta < 0.9 \). The second type of influence is considered as stabilizing the indoor air parameters variation. It is associated with the \( \beta \) in the range \( 0.9 \leq \beta \leq 1.1 \). The third type of influence is identified as promoting the variation of indoor air parameters. The respective range of \( \beta \) is \( \beta > 1.1 \).

In order to increase the soundness of the sample MSD analysis, which results in parameter \( \beta \) calculations, we additionally examined the root mean square error (RMSE) between the empirical sample MSD and fitted model. Namely, we can analyze the following error:

\[
RMSE = \sqrt{\frac{1}{K} \sum_{i=1}^{K} (\log(M_{N}(\tau_i)) - \log(\tau_i^{\beta}))^2,}
\]

where \( K \) is the number of time lags \( \tau_1, \tau_2, \ldots, \tau_K \) taken into account when fitting power-law function to sample MSD.

### 3. Experimental

Measurement data were obtained from quasi-continuous indoor air monitoring. In order to perform this type of measurements, wireless sensor technology was applied. The used equipment offered the following measurement resolution: temperature 0.1 °C, relative humidity 0.1%, and CO\(_2\) concentration 1 ppm. The measurement resolution of sensor devices...
directly determined the resolution of our MSD analysis, because it focused on the increments of indoor air parameters. The quantities were recorded with the time resolution of 15 s. This value was adjusted considering sensors inertia and the characteristics of electrical circuit in the sensor device. The measurement data, presented in the top panels of Figs. 2–4, were collected in the form of the univariate time series.

The data analyzed in this work were collected during 1.5 month of indoor air observation (from 18th April 2012 to 29th May, 2012), which extended over spring in temperate climate. The presented study was restricted to the conditions which are characteristic for a lecture room. In the measurement period, lecture room was in normal use. Single sensor device was located in its central part, at the height of about 1m. In the monitored space the air exchange was accomplished mainly by natural ventilation. On colder days, the heating system was in operation.
4. Results and discussion

The method introduced in this work was applied to characterize the collective impact of factors on the variation of three indicative parameters of IAQ (temperature, relative humidity and \( \text{CO}_2 \) concentration) in a lecture room. We emphasize that although the analysis of collective impact of factors on IAQ is performed separately for every of the above-mentioned indicative parameters, the conclusions about the variation of these parameters can be similar.

The data presented in top panels of Figs. 2–4 are the respective parameters of IAQ recorded over time interval 09:00–10:00 a.m. within consecutive days of time period 18th April 2012–29th May, 2012. There we observe some crucial changes in data values, which comes from the fact the recorded quantities may differ a lot over two consecutive days. Nevertheless we observe also significant changes of recorded quantities within time interval 09:00–10:00 in every single day. The second top panels of Figs. 2–4 present the increments of recorded quantities only within time interval 09:00–10:00 as we do not analyze the changes of data between consecutive days.

First, the applicability of sample MSD analysis was checked by testing the differentiated data for stationarity (one can find the differentiated data in the second top panels in Figs. 2–4). The required stationarity check was based on the autocorrelation and partial autocorrelation function of the differentiated time series of indoor air parameters. The plots, which demonstrate the typically obtained ACF and PACF of the analyzed data are displayed in two bottom panels in Figs. 2–4. One can observe that both ACF and PACF for \( T \), \( RH \) and \( \text{CO}_2 \) show the same behavior, namely their values tend to 0 for large values of lags. From this property we can expect that the increments of inspected time series are stationary. It is worth noticing that ACF and PACF in the case of \( T \) and \( RH \) tend to zero values quite quickly (for short lags), while for \( \text{CO}_2 \) they tend to 0 more slowly — indicating some possible differences between those quantities.

Second, the applicability of sample MSD time dependence analysis in terms of \( \beta \) was examined. The check was based on the quality of fitting with (4) when using data sets of various sizes taken from one sample. The plots shown in Figs. 5–7 refer to the time period 09:00–10:00 a.m. on working days. In this time span the data properties diverged most from the required ones. However, the generally linear character of the relationships observed in Figs. 5–7 validated the use of \( \beta \) as the descriptor of the sample MSD time dependence in the case of the analyzed indoor air parameters. Additionally, very small variation of the fitting lines slope, while downsizing the data set from 6223 to 2223 records, indicated the consistency of describing the entire hourly data sets with a single \( \beta \)’s.

Plots of \( \beta \), representing different types of collective impact of factors on temperature, relative humidity and \( \text{CO}_2 \) concentration variation, are shown in top panels of Figs. 8–10. The \( \beta \) was plotted versus the hour of the day in two groups of days: working days and days off. In the bottom panels of Figs. 8–10 we presented the RMSE (see Eq. (5)) associated with calculations of \( \beta \) from top panels. One can observe that the values of errors for three types of data are comparable and do not vary substantially over consecutive hours of a day, indicating that possible differences or similarities between the values of estimated \( \beta \)’s on working days and days off can be recognized as valid.

As shown in Figs. 8–10, at night-time \( \beta \) generally remained in a range of low values (\( \beta < 0.9 \)). This regularity was observed for all indoor air parameters: temperature (Fig. 8), relative humidity (Fig. 9) as well as \( \text{CO}_2 \) concentration (Fig. 10).
Low values of $\beta$ indicated that at night various factors jointly inhibited the temporal variation of indoor air parameters. Their influence had a retarding character. This kind of effect could result from indoor as well as ambient conditions, which are specific for the nighttime. In the first group, one may consider lack of human presence as the most important factor. It is actually synonymous with a lack of indoor, time varying $\text{CO}_2$, heat and water vapor sources. From the outdoor perspective there could be mentioned the lack sunshine. When present, it causes ambient air temperature changes in time and triggers airflows in the vicinity of the building walls. Without it, well established air infiltration and natural ventilation rate would have the impeding effect on indoor air. Interestingly, in the case of relative humidity, $\beta$ remained in the low range of values ($\beta < 0.9$) whole day, irrespective of the kind of day (Fig. 9). We may conclude that in the monitored space, factors influencing indoor air had retarding impact on humidity in the entire period of study. The influence promoting the variation of indoor air parameters was noticed only in case of $\text{CO}_2$ concentration and for temperature. This type of effect was indicated by $\beta$ taking up high values ($\beta > 1.1$). Such a situation was typically observed in daytime, predominantly on working days. The $\beta$ for $\text{CO}_2$ concentration, was very high (1.8) nearly entire daytime of working days (7 a.m.–7 p.m.). On days off, the impact that promoted changes of this contaminant concentration occurred shorter (11 a.m.–4 p.m.) and it was weaker ($\beta$ about 1.2). The influence promoting the variation of temperature was observed just on working days (see Fig. 8). It took place approximately from 8 a.m. to 7 p.m., similarly as for $\text{CO}_2$ concentration. However, in the case of temperature the effect was not so strong as for $\text{CO}_2$ concentration, as shown by smaller $\beta = 1.1–1.5$. We associate the promoting impact on indoor air parameters variation mainly with the presence of humans, acting as strong $\text{CO}_2$ and heat sources. Even their occasional presence during days off had a triggering effect on $\text{CO}_2$ concentration change. The other major factor possibly
Fig. 8. The $\beta$ parameter for temperature on distinct hours of day in the study period (top panel) and RMSE between the empirical sample MSD and fitted model (bottom panel).

Fig. 9. The $\beta$ parameter for relative humidity on distinct hours of day in the study period (top panel) and RMSE between the empirical sample MSD and fitted model (bottom panel).
involved was the sun radiation. Due to this factor a whole range of processes are activated which enhance the changes of indoor temperature, and not exclusively. However, this effect is restricted to the daytime.

The influence that stabilized the variation of indoor air parameters was indicated by $\beta$ in the range of values $0.9 \leq \beta \leq 1.1$. Based on top panels of Figs. 8 and 10, the period of the stabilizing impact may be viewed as a transition stage between the retarding and promoting one. While the last two regimes usually reigned over several subsequent hours of the day, the stabilizing influence was observed for two hours in turn, utmost (Figs. 8 and 10, working days). Counterintuitively, the influence which stabilized the variation of indoor air parameters could be related to the on-rise or decline of major factors which acted indoors and/or outdoors. It is shown by the time of day when this type of impact took place. Indoors, the appearance and disappearance of humans would be most important. From the outdoor perspective, the change between conditions typical for the daytime and nighttime could play the main role. The exceptionally long period (several hours) of the stabilizing influence was observed in the case of temperature variation on days off (Fig. 8). Considering that on working days the promoting effect was noted, we inferred that the major factor which enhanced indoor temperature changes was the human presence. Without it, the combined influence of factors could at most stabilize the temperature variation.

It is worth emphasizing that above presented analysis concerning variation of indicative parameters (temperature, relative humidity and CO$_2$ concentration) of IAQ is based on calculations of sample MSD for single trajectories covering time period of approximately 1.5 month. For the same reason at this stage of our research we cannot check the ergodicity of examined processes [50]. That will be possible when our data set will consist of the larger number of trajectories over examined time period or the trajectories for different calendar years over examined time period.

5. Conclusions

This work presents an original method for characterizing a collective influence of factors on the variation of indoor air parameters. It allows us to distinguish three types of influence: promoting, retarding and stabilizing their variation. The characterization is based on the time series of measurement data. There is utilized sample mean square displacement analysis.

The proposed method was applied to characterize the collective impact of factors on the variation of parameters that are descriptive for indoor air quality. Based on the obtained results, CO$_2$ and temperature variations were affected by all types of influences. In the case of relative humidity, exclusively the retarding effect was observed. The categories of impact could be correlated with the kind of day and time during the day. They represent configurations of factors causing influence of a particular type. On days off, the influence which retarded and stabilized the variation of indoor air parameters was
predominant. During working days there increased the contribution of the promoting effect. In the daytime, all types of influences could be encountered, but thenighttime was exclusively associated with the impacts which retarded the variation of indoor air parameters. In our opinion the introduced method may be useful for examining susceptibility of building environment to a wide range of factors.

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