List 1 Applied Logic
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1. Is it true that

a) $\models (p \to q) \leftrightarrow (\neg p \vee q)$,
b) $\models (p \wedge q) \to (p \leftrightarrow q)$,
c) $\models \neg(p \vee q) \leftrightarrow (\neg p \wedge \neg q)$,
d) $\models (p \to (q \vee r)) \leftrightarrow ((p \to q) \wedge (p \to r))$,
e) $\models (p \wedge (q \vee r)) \leftrightarrow ((p \wedge q) \vee (p \wedge r))$,
f) $\models (p \vee (q \wedge r)) \leftrightarrow ((p \wedge q) \vee (p \wedge r))$.

2. Show that every sentence (of propositional logic) is equivalent to a sentence in

a) conjunctive normal form,
b) disjunctive normal form.

3. Is it true that every sentence is equivalent to a sentence which has only two types of connectives: $\to, \neg$?

4. Is it true that

a) $\{p \to q\} \models p \vee q$,
b) $\{p, p \to q, \neg q \vee r\} \models p \wedge q \wedge r$,
c) $\{p \to q, \neg q \vee r\} \models p \wedge q \wedge r$,
d) $\{p \vee q, \neg p \vee r\} \models q \vee r$,
e) $\{p \vee \neg p\} \models \neg q \to q$
f) $\{p, \neg p\} \models \neg q \to q$.

5. Show that $\{\varphi_0, \varphi_1, \ldots, \varphi_n\} \models \psi$ is equivalent to $\models (\varphi_0 \wedge \varphi_1 \wedge \ldots \wedge \varphi_n) \to \psi$.

6. Let $\{C_1, C_2, \ldots, C_k\}$ be a set of clauses closed under taking resolvents. Let p be any propositional variable. Let us consider the following set of clauses

$$\{C_i \setminus \{p\} : \ i = 1, \ldots, k \text{ and } \neg p \notin C_i\}.$$

Show that it is closed under taking resolvents.

7. Prove that the resolution method is complete, i.e. if we didn't obtain $\Box$ then the set of clauses is satisfiable.

8. Using the resolution method decide which sets of clauses is satisfiable:

a) $\{p, \bar{p}qr, \bar{q}s, \bar{s}\}$,
b) $\{p, \bar{p}qr, \bar{q}s, \bar{s}, \bar{p}\bar{q}s\}$,
c) $\{pq, \bar{p}\bar{q}r, \bar{r}\}$,
d) $\{pq, \bar{p}q, p\bar{q}, \bar{p}\bar{q}\}$,
e) $\{\bar{p}, \bar{p}qr, \bar{q}s, \bar{s}\}$,
f) $\{pq, \bar{p}q, \bar{q}s, \bar{s}, \bar{p}\bar{q}s\}$.

If the set is satisfiable find a valuation showing it.

9. Using the resolution method decide which sentences are tautologies:

a) $((p \to q) \to p)$,
b) $((p \leftrightarrow q) \leftrightarrow p)$,
c) $((p \to q) \to r) \leftrightarrow (p \to (q \to r))$,
d) $p \vee (q \wedge r) \leftrightarrow (p \vee q) \wedge (p \vee r)$.

Use negation.